

AN INTEGRATED DECISION SUPPORT SYSTEM FOR STOCK INVESTMENT BASED ON SPHERICAL FUZZY PT-EDAS METHOD AND MEREC

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Abstract. The stock investment selection could be deemed as a classic multiple attribute group decision making (MAGDM) problem involving multiple conflicts and interleaved qualitative and quantitative attributes. Spherical fuzzy sets (SFSs) can excavate the potential vagueness and intricacy in MAGDM more effectively and deeply. This article we propose an integrated decision support system (IDSS) based on SFSs, prospect theory (PT), distance from average solution (EDAS) method and the Method based on the Removal Effects of Criteria (MEREC). The proposed IDSS, called SF-PT-EDAS-MEREC model, uses SFSs to describe the uncertain and obscure assessment information of DMs. The combination of PT and EDAS (PT-EDAS) method adequately captures DMs' psychological behavior characteristics to execute more reasonable alternative evaluation. The MEREC is utilized to efficaciously obtain unknown attribute weights. In addition, this paper also presents a novel score function to compare spherical fuzzy numbers (SFNs) more directly and efficiently. Eventually, in order to illustrate the practicability of the proposed IDSS, two numerical examples of stock investment selection are employed to achieve this. Meanwhile, the comparative study with existing approach further demonstrates the effectiveness and superiority of SF-PT-EDAS-MEREC model.

Keywords: spherical fuzzy sets, EDAS method, multiple attribute group decision making, prospect theory, MEREC, stock investment selection.

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Introduction

The rapid development of economy makes people's idle capital increase continuously, more and more people begin to pursue other investment ways besides savings, stock investment has become one of the primary choices for people's investment and financial management. As a complex financial activity, stock investment selection often involves multiple experts and needs to consider the constraints of multiple conflicting factors in the investment decision-making process. Effective stock investment decisions can maximize investment returns under certain investment risk conditions and promote the healthy and stable development of stock market. As a classical MAGDM problem, stock investment selection has always been a research hotspot in the decision-making territory. However, due to the complexity of the decision-making environment, the incompleteness of information and the fuzziness of human cognition, effective acquisition of evaluation information is the key to stock investment decision-making. In most of the existing literatures on stock investment decision making (Albadvi et al., 2007; Chen, 2018; Huang et al., 2004; Tiryaki & Ahlatcioglu, 2005; Zhao et al., 2021), researchers often use some tools such as accurate numerical values, fuzzy sets (FSs) (Zadeh, 1965), intuitionistic FSs (IFSs) (Atanassov, 1986), Pythagorean FSs (PYFSs) (Yager, 2014) to express DMs' judgments. But these tools may lead to the loss of some evaluation information, for example, accurate numerical values cannot detect the potential uncertainty of things, and FSs only use membership degree (MD) $\tilde{\phi}_{\bar{F}}(\bar{y})$ to reflect the preference of DMs. Although IFSs and PYFSs add non-membership degree (N-MD) $\tilde{\sigma}_{\bar{F}}(\bar{y})$ on the basis of FSs, they cannot directly express DMs' hesitation degree (HD) $\tilde{\rho}_{\bar{F}}(\bar{y})$, and the potential HD needs to depend on MD and N-MD rather than DMs' opinions. These bring a certain deviation to the full acquisition of evaluation information.

As a new improved form of IFSs and PYFSs, spherical fuzzy sets (SFSs) (Ashraf et al., 2019; Gundogdu & Kahraman, 2019; Mahmood et al., 2019) are capable of expressing DMs' opinions more comprehensively from four aspects: MD, N-MD, HD and rejection degree (RD). In SFSs, the MD, N-MD and HD are independent from each other and can be given freely by DMs in the range of 0 to 1. In addition, SFSs satisfy the condition: $0 \leq \tilde{\phi}_{\bar{F}}^2(\bar{y}) + \tilde{\sigma}_{\bar{F}}^2(\bar{y}) + \tilde{\rho}_{\bar{F}}^2(\bar{y}) \leq 1$ and provide DMs with wider and freer space. By far, SFSs have attracted the attention of many researchers in dealing with uncertain problems. For example, Kahraman et al. (2022) designed a new CRITIC method for prioritization of supplier selection criteria. Buyuk and Temur (2022) used spherical fuzzy AHP to assess food waste treatment option. H. Zhang et al. (2021) combined cumulative prospect theory (CPT) and MA-BAC approach under SFSs on green supplier selection. Farrokhizadeh, et al. (2021) integrated maximizing deviation and TOPSIS approach for advertising strategy selection problem in SFSs. H. Zhang et al. (2022a) presented SF-GRA based on CPT for emergency supplies supplier selection. Gundogdu (2020) extended MULTIMOORA approach to SFSs in settling multi-attribute decision-making (MADM) issues. H. Zhang et al. (2022c) integrated TOPSIS and CPT for residential location under SFSs. H. Zhang et al. (2022b) developed Dombi power Heronian mean aggregation operators in SFSs for MAGDM. Wei et al. (2019) established similarity measures by cosine function in SFSs. Nguyen et al. (2022) designed spherical fuzzy WASPAS approach based on entropy for international payment selection. Seyfi-Shishavan

et al. (2021) presented bi-objective linear assignment method based on SFSSs about insurance options choice. Zhang and Wei (2023) proposed a spherical fuzzy CPT-CoCoSo and D-CRITIC method for location selection of electric vehicles charging stations. In view of the advantages of SFSSs and their wide application in decision-making field, this article utilizes SFSSs to adequately express DMs' evaluation information in stock investment selection.

Up to now, a large number of MADM methods have been established in dealing with uncertain problems such as TOPSIS method (W. Su et al., 2022; Wang & Elhag, 2006; Yang et al., 2022), VIKOR method (Opricovic & Tzeng, 2004), PROMETHEE method (Brans et al., 1986), TODIM method (Gomes & Rangel, 2009) and BWM method (Rezaei, 2015) etc. Nevertheless, none of the aforementioned methods takes into account the distances between alternatives and average solutions. EDAS as a new and effective MADM approach was initiated by Keshavarz Ghorabae et al. (2015). This technique obtains the best alternative by measuring the positive and negative distances from the optimal amount which is computed with average solutions. On account of its stability, efficiency and simple operation process, EDAS method has been extended in abundance in recent years. Keshavarz Ghorabae et al. (2017) established EDAS model based on type-2 FSs for supplier assessment and order allocation. Kahraman, et al. (2017) utilized EDAS method for location selection of solid waste treatment. He et al. (2019) designed probabilistic uncertain linguistic EDAS for green supplier selection. X. Li et al. (2019) presented EDAS method for MAGDM in picture fuzzy sets (PFSs). Wei et al. (2021) established EDAS approach to probabilistic linguistic MAGDM. Ozcelik and Nalkiran (2021) introduced a new EDAS method based on trapezoidal bipolar FSs for healthcare system. Stanujkic et al. (2021) presented single-valued neutrosophic EDAS method to MADM. Menekse and Akdag (2022) established AHP and EDAS model based on SFSSs for distance education tool selection. N. Zhang et al. (2022d) proposed an evaluation and selection model of community group purchase platform based on WEPLPA-CPT-EDAS approach by combination of the weighted probabilistic linguistic power average operator, CPT and EDAS method.

However, most of the above evaluation processes often assume that DMs make decisions under completely rational conditions and fail to take into account their subjective risk appetite. As a descriptive decision model of risk preference, PT (Kahneman & Tversky, 1979) can efficaciously reflect the DMs' mental characteristics for facing gains and losses. Currently, the integration of PT and some MADM approaches to solve practical issues has become a new study hotspot in decision system. For example, Chen et al. (2020) proposed a multi-stage decision model based on PT and PROMETHEE method for renewable energy options. Liu and Zhang (2021) designed an improved MABAC approach based on PT and CCSD in normal wiggly hesitant FSs. Tian et al. (2022) utilized PT to establish an extended MULTIMOORA model in PFSs for medical institution selection. Jiang et al. (2022) presented MABAC approach based on PT under PFSs. Jia and Wang (2020) developed rough-number-based MAGDM approach via combining the BWM and PT. Fan et al. (2022) presented PT-MARCOS method for settling MADM under neutrosophic cubic environment. Huang et al. (2021) presented an enhanced EDAS approach combined with PT in real number environment. Y. Su et al. (2022) fully exploited the merits of EDAS and PT to propose PT-EDAS model for probabilistic uncertain linguistic MAGDM issue. P. Li et al. (2022) integrated EDAS approach

and PT for highway investment project selection in PYFSs. But so far, there are few researches on spherical fuzzy MAGDM based on PT at home and abroad. More importantly, PT-EDAS approach has not been extended in SFSs to settle MAGDM issues.

Attribute weights are important factors in affecting decision results. For MAGDM, there are usually three ways to obtain attribute weights, which are subjective weighting methods by DMs' judgement, objective weighting methods from the initial assessment information and the hybrid weight methods through incorporation of subjective and objective weights. Subjective weighting methods (such as direct giving, AHP, SWARA, KEMIRA and FUCOM etc.) mainly depend on DMs' personal preference information, but different DMs may have different preferences for attribute sets. Therefore, subjective weighting methods tend to produce strong subjective preference, which lead to certain deviation in the effective acquisition of weight information. To surmount the subjectivity and improve the rationality of attribute weights, objective weighting methods become very popular in the evaluation process. Because they do not require any subjective judgment of DMs but based on the initial evaluation information or decision matrix to capture the performance of attributes with some specific calculation algorithms for obtaining weight information. The commonly used objective weight methods include entropy method (Deng et al., 2000), the maximizing deviation method (Wu & Chen, 2007), CRITIC method (Diakoulaki et al., 1995), etc. As a novel objective weight calculation method, MEREC was proposed by Keshavarz-Ghorabae et al. (2021). Compare with the aforementioned objective weighting methods, MEREC can efficaciously obtain attribute weight information by measuring the removal effect of each attribute on the overall performance of alternatives. In this method, when the removal of an attribute produces a greater impact on the overall performance of alternatives, it will be assigned a higher weight. In view of the variations, MEREC provides a new perspective for obtaining attribute weights by removing attributes to measure the performances of alternatives to enhance the robustness of the results. In addition, the calculation process of MEREC is simple and the operation is flexible because DMs can use different functions to determine the performances. At present, MEREC has been successfully applied to some practical problems (Keshavarz-Ghorabae et al., 2021; Rani et al., 2022; Trung & Thinh, 2021). In consideration of its virtues, this article will extend MEREC to SFSs to effectively obtain unknown attribute weights.

By the above statement, the motivations for this article are as follows: (1) Compared with other FSs, SFSs can depict the uncertainty and fuzziness of things deeply and effectively. (2) Although EDAS is a useful tool to MAGDM problems, EDAS often assumes that DMs are absolute rationality in most decision-making processes without considering their risk preference. PT can efficaciously reflect DMs' mental behavior when facing the risks. Currently, the integration of PT and some decision approaches for resolving MAGDM issues has become a new study topic of decision system. However, considering the risk appetite of DMs, the combination of EDAS and PT for dealing with MAGDM in SFSs has not been developed yet. (3) As a latest and effective attribute weight calculation method, MEREC has not received attention and application for solving spherical fuzzy MAGDM. (4) As one of the research hotspots of MAGDM, scientific and reasonable stock investment selection can help enterprises or individuals reduce investment risks and obtain higher returns, which have important research value. However, most of the existing researches on stock investment selection are carried

out under the condition that DMs are perfectly rational. Moreover, the use of other tools to express MDs' assessment information on stock investment is not comprehensive enough. Follow the reasons above, this paper combines SFSs, PT, EDAS method and MEREC to develop an IDSS for dealing with uncertain problems. Moreover, a novel score function is presented for comparing the sizes of SFNs more efficaciously. At last, the presented IDSS is utilized for the problem of stock investment selection to demonstrate its practicality and legality.

The primary contributions for this article are listed as follows: (1) A new score function is presented and its some properties are discussed to measure the sizes of SFNs more efficaciously. (2) MEREC is extended to acquire unknown attribute weights in spherical fuzzy environment. (3) The traditional EDAS method is improved by integrating EDAS method and PT to capture the mental sense of MDs. (4) An IDSS is developed for settling MAGDM issues. (5) SF-PT-EDAS-MEREC model is used in stock investment selection to elucidate the practicability for the presented IDSS and comparison research is rendered to further certify the effectiveness and meliority for SF-PT-EDAS-MEREC model. (6) The proposed IDSS offers DMs more choices for resolving MAGDM issues and affords some valuable references in promoting the further expansion about PT-EDAS approach in other decision environments.

The rest of this article is arranged as follows: The fundamental knowledge of SFSs as well as the basic thoughts of MEREC, EDAS method and PT are elaborated in part 1. A novel score function is proposed in part 2. SF-PT-EDAS-MEREC model is developed in part 3. The proposed IDSS is utilize for stock investment selection to testify its practicability in part 4. Meanwhile, the effectiveness of SF-PT-EDAS-MEREC model is proved by existing approaches in SFSs. Ultimately, we briefly summarize this article.

1. Preliminaries

1.1. SFNs

In this chapter, some primary components of SFSs as well as the basic thoughts of MEREC, EDAS method and PT will be depicted.

Definition 1 (Gundogdu & Kahraman, 2019). Let \tilde{A} be a nonempty fixed set, then SFS Γ on \tilde{A} is defined as:

$$\tilde{\Gamma} = \{ \langle \tilde{a}, (\tilde{\phi}_{\tilde{F}}(\tilde{a}), \tilde{\sigma}_{\tilde{F}}(\tilde{a}), \tilde{\rho}_{\tilde{F}}(\tilde{a})) \mid \tilde{a} \in \tilde{A} \}, \tag{1}$$

where $\tilde{\phi}_{\tilde{F}} : \tilde{A} \rightarrow [0,1], \tilde{\sigma}_{\tilde{F}} : \tilde{A} \rightarrow [0,1], \tilde{\rho}_{\tilde{F}} : \tilde{A} \rightarrow [0,1]$ and $0 \leq \tilde{\phi}_{\tilde{F}}^2(\tilde{a}) + \tilde{\sigma}_{\tilde{F}}^2(\tilde{a}) + \tilde{\rho}_{\tilde{F}}^2(\tilde{a}) \leq 1, \forall \tilde{a} \in \tilde{A}$. At the same time, for each \tilde{a} , the numbers $\tilde{\phi}_{\tilde{F}}(\tilde{a}), \tilde{\sigma}_{\tilde{F}}(\tilde{a})$ and $\tilde{\rho}_{\tilde{F}}(\tilde{a})$ are MD, N-MD and HD of \tilde{a} to $\tilde{\Gamma}$ respectively. $\tilde{\omega}_{\tilde{F}}(\tilde{a}) = \sqrt{1 - \tilde{\phi}_{\tilde{F}}^2(\tilde{a}) - \tilde{\sigma}_{\tilde{F}}^2(\tilde{a}) - \tilde{\rho}_{\tilde{F}}^2(\tilde{a})}$ denotes the RD. The triplet $\tilde{\Gamma}_1 = (\tilde{\phi}, \tilde{\sigma}, \tilde{\rho})$ is known as SFN.

In the following, some operational laws of SFNs are introduced as follows:

Definition 2 (Gundogdu & Kahraman, 2019). Let $\tilde{\Gamma}_1 = (\tilde{\phi}_1, \tilde{\sigma}_1, \tilde{\rho}_1)$ and $\tilde{\Gamma}_2 = (\tilde{\phi}_2, \tilde{\sigma}_2, \tilde{\rho}_2)$ be two SFNs respectively, then:

$$i. (\tilde{\Gamma}_1)^c = (\tilde{\sigma}_1, \tilde{\phi}_1, \tilde{\rho}_1) \text{ (where } (\tilde{\Gamma}_1)^c \text{ denotes the complement of } \tilde{\Gamma}_1 \text{);} \tag{2}$$

$$\text{ii. } \tilde{\Gamma}_1 \oplus \tilde{\Gamma}_2 = \left(\begin{array}{cc} \left(\tilde{\phi}_1^2 + \tilde{\phi}_2^2 - \tilde{\phi}_1^2 \tilde{\phi}_2^2 \right)^{1/2}, & \tilde{\sigma}_1 \tilde{\sigma}_2, \\ \left((1 - \tilde{\phi}_2^2) \tilde{\rho}_1^2 + (1 - \tilde{\phi}_1^2) \tilde{\rho}_2^2 - \tilde{\rho}_1^2 \tilde{\rho}_2^2 \right)^{1/2} \end{array} \right); \tag{3}$$

$$\text{iii. } \tilde{\Gamma}_1 \otimes \tilde{\Gamma}_2 = \left(\begin{array}{cc} \tilde{\phi}_1 \tilde{\phi}_2, & \left(\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2 - \tilde{\sigma}_1^2 \tilde{\sigma}_2^2 \right)^{1/2}, \\ \left((1 - \tilde{\sigma}_2^2) \tilde{\rho}_1^2 + (1 - \tilde{\sigma}_1^2) \tilde{\rho}_2^2 - \tilde{\rho}_1^2 \tilde{\rho}_2^2 \right)^{1/2} \end{array} \right); \tag{4}$$

$$\text{iv. } \theta \cdot \tilde{\Gamma}_1 = \left(\left(1 - (1 - \tilde{\phi}_1^2)^\theta \right)^{1/2}, \tilde{\sigma}_1^\theta, \left((1 - \tilde{\phi}_1^2)^\theta - (1 - \tilde{\phi}_1^2 - \tilde{\rho}_1^2)^\theta \right) \right), \quad \theta > 0; \tag{5}$$

$$\text{v. } \tilde{\Gamma}_1^\theta = \left(\tilde{\phi}_1^\theta, \left(1 - (1 - \tilde{\sigma}_1^2)^\theta \right)^{1/2}, \left((1 - \tilde{\sigma}_1^2)^\theta - (1 - \tilde{\sigma}_1^2 - \tilde{\rho}_1^2)^\theta \right)^{1/2} \right), \quad \theta > 0. \tag{6}$$

Definition 3 (Gundogdu & Kahraman, 2019; Sharaf, 2021). Let $\tilde{\Gamma}_1 = (\tilde{\phi}_1, \tilde{\sigma}_1, \tilde{\rho}_1)$ be an SFN, then the score function $S''(\tilde{\Gamma}_1)$ and the accuracy function $A''(\tilde{\Gamma}_1)$ are given as:

$$S''(\tilde{\Gamma}_1) = (\tilde{\phi}_1 - \tilde{\rho}_1)^2 - (\tilde{\sigma}_1 - \tilde{\rho}_1)^2; \tag{7}$$

$$A''(\tilde{\Gamma}_1) = \tilde{\phi}_1^2 + \tilde{\sigma}_1^2 + \tilde{\rho}_1^2. \tag{8}$$

In addition, for any two SFNs $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$, note that: $\tilde{\Gamma}_1 < \tilde{\Gamma}_2$ if and only if

- i. $S''(\tilde{\Gamma}_1) < S''(\tilde{\Gamma}_2)$ or
- ii. $S''(\tilde{\Gamma}_1) = S''(\tilde{\Gamma}_2)$ and $A''(\tilde{\Gamma}_1) < A''(\tilde{\Gamma}_2)$.

Furthermore, in (Ashraf et al., 2019), the comparison rules of SFNs are given as follows:

Definition 4 (Ashraf et al., 2019). Let $\tilde{\Gamma}_1 = (\tilde{\phi}_1, \tilde{\sigma}_1, \tilde{\rho}_1)$ be an SFN, then the score function $S''(\tilde{\Gamma}_1)$, accuracy function $A''(\tilde{\Gamma}_1)$ as well as certainty function $C''(\tilde{\Gamma}_1)$ of $\tilde{\Gamma}_1$ are given as:

$$S''(\tilde{\Gamma}_1) = \frac{1}{3}(2 + \tilde{\phi}_1 - \tilde{\sigma}_1 - \tilde{\rho}_1); \tag{9}$$

$$A''(\tilde{\Gamma}_1) = \tilde{\phi}_1 - \tilde{\sigma}_1; \tag{10}$$

$$C''(\tilde{\Gamma}_1) = \tilde{\phi}_1. \tag{11}$$

Note that: $\tilde{\Gamma}_1 < \tilde{\Gamma}_2$ if and only if

- i. $S''(\tilde{\Gamma}_1) < S''(\tilde{\Gamma}_2)$ or
- ii. $S''(\tilde{\Gamma}_1) = S''(\tilde{\Gamma}_2)$ and $A''(\tilde{\Gamma}_1) < A''(\tilde{\Gamma}_2)$ or
- iii. $S''(\tilde{\Gamma}_1) = S''(\tilde{\Gamma}_2)$, $A''(\tilde{\Gamma}_1) = A''(\tilde{\Gamma}_2)$ and $C''(\tilde{\Gamma}_1) < C''(\tilde{\Gamma}_2)$.

Definition 5 (Kutlu Gündoğdu & Kahraman, 2021; Zhang & Xu, 2014). Let $\tilde{\Gamma}_1 = (\tilde{\phi}_1, \tilde{\sigma}_1, \tilde{\rho}_1)$ and $\tilde{\Gamma}_2 = (\tilde{\phi}_2, \tilde{\sigma}_2, \tilde{\rho}_2)$ represent two SFNs respectively, the distance between $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ is given as:

$$d(\tilde{\Gamma}_1, \tilde{\Gamma}_2) = \frac{1}{2} \left(\left| \tilde{\phi}_1^2 - \tilde{\phi}_2^2 \right| + \left| \tilde{\sigma}_1^2 - \tilde{\sigma}_2^2 \right| + \left| \tilde{\rho}_1^2 - \tilde{\rho}_2^2 \right| \right). \tag{12}$$

Definition 6 (Gundogdu & Kahraman, 2019). Let $\tilde{\Gamma}_h = (\tilde{\phi}_h, \tilde{\sigma}_h, \tilde{\rho}_h) (h = 1, 2, \dots, t)$ be a set of SFNs and $\omega = (\omega_1, \omega_2, \dots, \omega_t)^T$ is the corresponding weight vector, with $\omega_h \in [0, 1], \sum_{h=1}^t \omega_h = 1$. Then spherical weighted arithmetic mean (SWAM) operator is defined as:

$$SWAM(\tilde{\Gamma}_1, \tilde{\Gamma}_2, \dots, \tilde{\Gamma}_t) = \sum_{h=1}^t \omega_h \tilde{\Gamma}_h = \left(\begin{array}{cc} [1 - \prod_{h=1}^t (1 - \tilde{\phi}_h^2)^{\omega_h}]^{1/2}, & \prod_{h=1}^t \tilde{\sigma}_h^{\omega_h}, \\ [\prod_{h=1}^t (1 - \tilde{\phi}_h^2)^{\omega_h} - \prod_{h=1}^t (1 - \tilde{\phi}_h^2 - \tilde{\rho}_h^2)^{\omega_h}]^{1/2} & \end{array} \right). \quad (13)$$

Definition 7 (Gundogdu & Kahraman, 2019). Let $\tilde{\Gamma}_h = (\tilde{\phi}_h, \tilde{\sigma}_h, \tilde{\rho}_h) (h = 1, 2, \dots, t)$ be a set of SFNs and $\omega = (\omega_1, \omega_2, \dots, \omega_t)^T$ is the corresponding weight vector, with $\omega_h \in [0, 1], \sum_{h=1}^t \omega_h = 1$. Then spherical weighted geometric mean (SWGGM) operator is defined as:

$$SWGGM(\tilde{\Gamma}_1, \tilde{\Gamma}_2, \dots, \tilde{\Gamma}_t) = \prod_{h=1}^t \tilde{\Gamma}_h^{\omega_h} = \left(\begin{array}{cc} \prod_{h=1}^t \tilde{\phi}_h^{\omega_h}, & [1 - \prod_{h=1}^t (1 - \tilde{\sigma}_h^2)^{\omega_h}]^{1/2}, \\ [\prod_{h=1}^t (1 - \tilde{\sigma}_h^2)^{\omega_h} - \prod_{h=1}^t (1 - \tilde{\sigma}_h^2 - \tilde{\rho}_h^2)^{\omega_h}]^{1/2} & \end{array} \right). \quad (14)$$

1.2. The MEREC

MEREC was introduced by Keshavarz-Ghorabae et al. (2021) in 2021. As a new method for objective weighting, it mainly measures removal influences of each attribute on the overall performance of alternatives to determine attribute weights. That is, when the deletion of an attribute brings about more impacts on the overall performance of alternatives, it will be given greater weight. The general procedures for MEREC are as below:

Step 1. Establish decision matrix.

For a MADM issue, assuming that the original decision matrix by DM is given as:

$$N = (n_{gh})_{s \times t} = \begin{bmatrix} n_{11} & n_{12} & \dots & n_{1t} \\ n_{21} & n_{22} & \dots & n_{2t} \\ \vdots & \vdots & \vdots & \vdots \\ n_{s1} & n_{s2} & \dots & n_{st} \end{bmatrix}_{s \times t}, \quad g = 1, 2, \dots, s; h = 1, 2, \dots, t. \quad (15)$$

Step 2. Standardize the decision matrix by Eq. (16):

$$n'_{gh} = \begin{cases} \frac{\min_g n_{gh}}{g}, & \text{for benefit attribute} \\ \frac{n_{gh}}{g}, & \\ \frac{n_{gh}}{\max_g n_{gh}}, & \text{for cost attribute} \end{cases}. \quad (16)$$

Step 3. Compute the overall performance for each alternative with Eq. (17):

$$Q_g = \ln \left(1 + \left(\frac{1}{t} \sum_{h=1}^t |\ln n'_{gh}| \right) \right), \quad g = 1, 2, \dots, s. \quad (17)$$

Step 4. Remove each attribute to get the performance of alternative based on Eq. (18):

$$\tilde{Q}_{gh} = \ln \left(1 + \left(\frac{1}{t} \sum_{j=1, j \neq h}^t |\ln n'_{gj}| \right) \right), \quad g = 1, 2, \dots, s; h = 1, 2, \dots, t, \tag{18}$$

where \tilde{Q}_{gh} indicates the overall performance for g th alternative as regards the removal of h th attribute.

Step 5. Determine the summation of absolute deviations for each attribute depending on Eq. (19):

$$SQ_h = \sum_{g=1}^s |\tilde{Q}_{gh} - Q_g|, \quad h = 1, 2, \dots, t. \tag{19}$$

Step 6. Get attribute weights by Eq. (20):

$$\tilde{\omega}_h = \frac{SQ_h}{\sum_{h=1}^t SQ_h}, \quad h = 1, 2, \dots, t, \tag{20}$$

where $\tilde{\omega}_h$ represents weight of h th attribute.

1.3. The classical EDAS method

The EDAS is a practical MADM method, which determines the distance of each alternative from the average values to get the best alternative. The main idea is displayed as below:

Suppose that the initial decision matrix is given by DM as shown in Eq. (15), and $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \dots, \tilde{\omega}_t)^T$ is the weight vector of attributes.

Step 1. Compute the average solution of each attribute with Eq. (21):

$$\bar{n}_h = \frac{\sum_{g=1}^s n_{gh}}{s}, \quad h = 1, 2, \dots, t. \tag{21}$$

Step 2. Construct positive distance from average (PDA) as well as negative distance from average (NDA).

Based on benefit attribute, the PDA and NDA are respectively defined as:

$$PDA_{gh} = \frac{\max(0, n_{gh} - \bar{n}_h)}{\bar{n}_h}, \quad g = 1, 2, \dots, s; h = 1, 2, \dots, t; \tag{22}$$

$$NDA_{gh} = \frac{\max(0, \bar{n}_h - n_{gh})}{\bar{n}_h}, \quad g = 1, 2, \dots, s; h = 1, 2, \dots, t. \tag{23}$$

Furthermore, for cost attribute, the PDA and NDA are respectively defined as:

$$PDA_{gh} = \frac{\max(0, \bar{n}_h - n_{gh})}{\bar{n}_h}, \quad g = 1, 2, \dots, s; h = 1, 2, \dots, t; \tag{24}$$

$$NDA_{gh} = \frac{\max(0, n_{gh} - \bar{n}_h)}{\bar{n}_h}, \quad g = 1, 2, \dots, s; h = 1, 2, \dots, t. \tag{25}$$

Step 3. Obtain the weighted PDA and weighted NDA for each alternative by Eqs (26)–(27):

$$WP_g = \sum_{h=1}^t PDA_{gh} \cdot \tilde{\omega}_h, \quad g = 1, 2, \dots, s; \tag{26}$$

$$WN_g = \sum_{h=1}^t NDA_{gh} \cdot \tilde{\omega}_h, \quad g = 1, 2, \dots, s. \tag{27}$$

Step 4. Standard the values about weighted PDA as well as weighted NDA depending on Eqs (28)–(29):

$$SP_g = \frac{WP_g}{\max_g WP_g}, \quad g = 1, 2, \dots, s; \tag{28}$$

$$SN_g = 1 - \frac{WN_g}{\max_g WN_g}, \quad g = 1, 2, \dots, s. \tag{29}$$

Step 5. Get the appraisal score for each alternative based on Eq. (30):

$$AS_g = \frac{1}{2}(SP_g + SN_g), \quad g = 1, 2, \dots, s. \tag{30}$$

Step 6. Rank all alternatives by appraisal scores in descending sort to get the preferable one.

1.4. Prospect theory

In 1979, Kahneman and Tversky (1979) presented the PT on the basis of bounded rationality to explain various phenomena that do not conform to the expected utility theory in the decision-making process. PT replaces utility and probability in traditional expected utility theory with value function and probability weight function, which can model people’s psychological behavior in making decisions and make the decision results more consistent with the inherent thinking habits of human beings. In PT, the prospect value \tilde{V} is composed of value function and probability weight function, that is

$$\tilde{V} = \sum_{i=1}^n \tilde{v}(\tilde{x}_i) \cdot \kappa(\tilde{e}_i), \tag{31}$$

where $\tilde{v}(\tilde{x}_i)$ is the value function determined with (32), which can fully reflect DMs’ risk attitude and subjective preference when facing gain and loss.

$$\tilde{v}(\tilde{x}_i) = \begin{cases} (\tilde{x}_i)^\alpha, & \tilde{x}_i \geq 0 \\ -\lambda(-\tilde{x}_i)^\beta, & \tilde{x}_i < 0 \end{cases}, \tag{32}$$

where \tilde{x}_i represents the difference with respect to the reference point, and $\tilde{x}_i \geq 0$ indicates the gain, whereas $\tilde{x}_i < 0$ means the loss. α, β are the risk attitude coefficients of DMs and mean the preference degrees in the region of gain and loss. λ denotes the coefficient of loss aversion that is more sensitive to loss than gain.

The probability weight function $\kappa(\tilde{e}_i)$ reflects that people often overestimate low probability events and underestimate high probability events in reality, which is calculated by Tversky and Kahneman (1992) as follow:

$$\kappa(\bar{\varepsilon}_i) = \begin{cases} \frac{\bar{\varepsilon}_i^\eta}{(\bar{\varepsilon}_i^\eta + (1 - \bar{\varepsilon}_i)^\eta)^\eta} \cdot \frac{1}{\delta}, & x_i \geq 0 \\ \frac{\bar{\varepsilon}_i^\delta}{(\bar{\varepsilon}_i^\delta + (1 - \bar{\varepsilon}_i)^\delta)^\delta} \cdot \frac{1}{\eta}, & x_i < 0 \end{cases}, \tag{33}$$

where $\bar{\varepsilon}_i$ denotes probability. η, δ express the curvature for probability weight function of gain as well as loss and reflect DMs' different risk attitudes towards gain and loss, respectively.

2. A new score function for SFNs

A new score function is introduced for measuring the size of SFNs in this chapter. Furthermore, we discuss some of its properties and illustrate the availability as well as superiority for the presented score function via existing score function.

Definition 8. Let $\tilde{\Gamma}_1 = (\tilde{\phi}_1, \tilde{\sigma}_1, \tilde{\rho}_1)$ be an SFN, then score function $S''(\tilde{\Gamma}_1)$ of $\tilde{\Gamma}_1$ is defined as:

$$S''(\tilde{\Gamma}_1) = \frac{(1 + \tilde{\phi}_1^2 - \tilde{\sigma}_1^2 - \tilde{\rho}_1^2) \cdot (\tilde{\phi}_1^2 + \tilde{\sigma}_1^2 + \tilde{\rho}_1^2)}{2}. \tag{34}$$

Furthermore, for any two SFNs $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$, we have:

- (1) If $S''(\tilde{\Gamma}_1) > S''(\tilde{\Gamma}_2)$, then $\tilde{\Gamma}_1 > \tilde{\Gamma}_2$;
- (2) If $S''(\tilde{\Gamma}_1) < S''(\tilde{\Gamma}_2)$, then $\tilde{\Gamma}_1 < \tilde{\Gamma}_2$;
- (2) If $S''(\tilde{\Gamma}_1) = S''(\tilde{\Gamma}_2)$, then $\tilde{\Gamma}_1 = \tilde{\Gamma}_2$.

Theorem 1. Let $\tilde{\Gamma}_1 = (\tilde{\phi}_1, \tilde{\sigma}_1, \tilde{\rho}_1)$ be an SFN, then new score function $S''(\tilde{\Gamma}_1)$ satisfies:

- (1) $S''(\tilde{\Gamma}_1) \in [0, 1]$;
- (2) $S''(\tilde{\Gamma}_1) = 1$ if and only if $\tilde{\phi}_1 = 1, \tilde{\sigma}_1 = 0, \tilde{\rho}_1 = 0$; $S''(\tilde{\Gamma}_1) = 0$ if and only if $\tilde{\phi}_1 = 0, \tilde{\sigma}_1 = 1, \tilde{\rho}_1 = 0$ or $\tilde{\phi}_1 = 0, \tilde{\sigma}_1 = 0, \tilde{\rho}_1 = 1$.

Proof.

(1) Let $\tilde{\Omega} = \frac{(1 + \tilde{\phi}_1^2 - \tilde{\sigma}_1^2 - \tilde{\rho}_1^2)}{2}$, then $\frac{\partial \tilde{\Omega}}{\partial \tilde{\phi}_1} = \tilde{\phi}_1, \frac{\partial \tilde{\Omega}}{\partial \tilde{\sigma}_1} = -\tilde{\sigma}_1, \frac{\partial \tilde{\Omega}}{\partial \tilde{\rho}_1} = -\tilde{\rho}_1$.

Since $0 \leq \tilde{\phi}_1, \tilde{\sigma}_1, \tilde{\rho}_1 \leq 1$,

then $\tilde{\Omega}$ is monotonically increasing with respect to $\tilde{\phi}_1$ and decreasing with respect to $\tilde{\sigma}_1$ and $\tilde{\rho}_1$ respectively.

Furthermore, based on $0 \leq \tilde{\phi}_1^2 + \tilde{\sigma}_1^2 + \tilde{\rho}_1^2 \leq 1$, so when $\tilde{\phi}_1 = 1, \tilde{\sigma}_1 = 0, \tilde{\rho}_1 = 0$, then $\tilde{\Omega}_{\max} = 1$, when $\tilde{\phi}_1 = 0, \tilde{\sigma}_1 = 1, \tilde{\rho}_1 = 0$ or $\tilde{\phi}_1 = 0, \tilde{\sigma}_1 = 0, \tilde{\rho}_1 = 1$ then $\tilde{\Omega}_{\min} = 0$, it follows that $0 \leq \tilde{\Omega} \leq 1$. Hence, $S''(\tilde{\Gamma}_1) = \tilde{\Omega}_1 \cdot (\tilde{\phi}_1^2 + \tilde{\sigma}_1^2 + \tilde{\rho}_1^2) \in [0, 1]$.

- (2) According to the proof of (1), (2) is obvious.

Compared with the score function in Definition 3 and Definition 4, the new score function is more direct in comparing the size of two SFNs because it does not need to rely on other auxiliary functions. For example, for SFNs $\tilde{\Gamma}_1 = (0.8, 0.4, 0.2), \tilde{\Gamma}_2 = (0.7, 0.3, 0.1)$ and $\tilde{\Gamma}_3 = (0.6, 0.2, 0.2)$, In Definition 3, since $S''(\tilde{\Gamma}_1) = (0.8 - 0.2)^2 - (0.4 - 0.2)^2 = 0.32, S''(\tilde{\Gamma}_2) = (0.7 - 0.1)^2 - (0.3 - 0.1)^2 = 0.32$, then $S''(\tilde{\Gamma}_1) = S''(\tilde{\Gamma}_2)$. Based on the comparison rules in Definition 3, there is need to sort two SFNs by means of accu-

racy function, and accuracy functions of $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ are: $A''(\tilde{\Gamma}_1) = 0.8^2 + 0.4^2 + 0.2^2 = 0.84$, $A''(\tilde{\Gamma}_2) = 0.7^2 + 0.3^2 + 0.1^2 = 0.59$, so $\tilde{\Gamma}_1 > \tilde{\Gamma}_2$. However, by Definition 8, $S''(\tilde{\Gamma}_1) = \frac{1}{2}(1 + 0.8^2 - 0.4^2 - 0.2^2)(0.8^2 + 0.4^2 + 0.2^2) = 0.6048$, $S''(\tilde{\Gamma}_2) = \frac{(1 + 0.7^2 - 0.3^2 - 0.1^2)(0.7^2 + 0.3^2 + 0.1^2)}{2} = 0.4101$, so $\tilde{\Gamma}_1 > \tilde{\Gamma}_2$. Furthermore, in Definition 4, since $S''(\tilde{\Gamma}_1) = \frac{1}{3}(2 + 0.8 - 0.4 - 0.2) = 0.73333$, $S''(\tilde{\Gamma}_3) = \frac{1}{3}(2 + 0.6 - 0.2 - 0.2) = 0.73333$, then $S''(\tilde{\Gamma}_1) = S''(\tilde{\Gamma}_3)$, and accuracy function of $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_3$ are: $A''(\tilde{\Gamma}_1) = 0.8 - 0.4 = 0.4$, $A''(\tilde{\Gamma}_3) = 0.6 - 0.2 = 0.4$, at this point, the size of $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_3$ can only be measured by certain functions: $C''(\tilde{\Gamma}_1) = 0.8 > C''(\tilde{\Gamma}_3) = 0.6$, so $\tilde{\Gamma}_1 > \tilde{\Gamma}_3$. However, by Definition 8, $S''(\tilde{\Gamma}_1) = \frac{(1 + 0.8^2 - 0.4^2 - 0.2^2)(0.8^2 + 0.4^2 + 0.2^2)}{2} = 0.6048$, $S''(\tilde{\Gamma}_3) = \frac{(1 + 0.6^2 - 0.2^2 - 0.2^2)(0.6^2 + 0.2^2 + 0.2^2)}{2} = 0.2816$, so $\tilde{\Gamma}_1 > \tilde{\Gamma}_3$.

3. SF-PT-EDAS-MEREC model for MAGDM problems

Let $ST = \{ST_1, ST_2, \dots, ST_s\}$ be a set of s alternatives, $SS = \{SS_1, SS_2, \dots, SS_t\}$ be a set of t attributes, and $EX = \{EX_1, EX_2, \dots, EX_p\}$ represents the collection of p experts. $\tilde{\zeta} = (\tilde{\zeta}_1, \tilde{\zeta}_2, \dots, \tilde{\zeta}_p)^T$ represents experts' weight vector, satisfying $\tilde{\zeta}_r \geq 0$ and $\sum_{r=1}^p \tilde{\zeta}_r = 1$, and attribute weights are unknown.

Furthermore, $\tilde{X}^r = (\tilde{x}_{gh}^r)_{s \times t} = (\tilde{\phi}_{gh}^r, \tilde{\sigma}_{gh}^r, \tilde{\rho}_{gh}^r)_{s \times t}$ denotes the decision information of the r th expert by using SFNs, $\tilde{\phi}_{gh}^r, \tilde{\sigma}_{gh}^r$ and $\tilde{\rho}_{gh}^r$ are MD, N-MD and HD about \tilde{x}_{gh}^r respectively, meeting the conditions: $\tilde{\phi}_{gh}^r, \tilde{\sigma}_{gh}^r, \tilde{\rho}_{gh}^r \in [0, 1]$, and $0 \leq (\tilde{\phi}_{gh}^r)^2 + (\tilde{\sigma}_{gh}^r)^2 + (\tilde{\rho}_{gh}^r)^2 \leq 1$ ($g = 1, 2, \dots, s, h = 1, 2, \dots, t, r = 1, 2, \dots, p$).

In the following we give the general process for SF-PT-EDAS-MEREC model (also shown in Figure 1).

Step 1. Obtain evaluation information from DMs with the linguistic terms shown in Table 1.

Step 2. Aggregate all DMs' evaluation information $\tilde{X}^r = (\tilde{x}_{gh}^r)_{s \times t}$ ($r = 1, 2, \dots, p$) to acquire group decision information $\tilde{Z} = (\tilde{z}_{gh})_{s \times t}$ by employing SWAM operator.

$$\tilde{z}_{gh} = (\tilde{\omega}_{gh}, \tilde{\gamma}_{gh}, \tilde{\phi}_{gh}) = \text{SWAM}_{\tilde{\zeta}}(\tilde{x}_{gh}^1, \tilde{x}_{gh}^2, \dots, \tilde{x}_{gh}^p) = \tilde{\zeta}_1 \tilde{x}_{gh}^1 + \tilde{\zeta}_2 \tilde{x}_{gh}^2 + \dots + \tilde{\zeta}_p \tilde{x}_{gh}^p = \left(\begin{array}{l} [1 - \prod_{r=1}^p (1 - (\tilde{\phi}_{gh}^r)^2)^{\tilde{\zeta}_r}]^{1/2}, \quad \prod_{r=1}^p (\tilde{\sigma}_{gh}^r)^{\tilde{\zeta}_r}, \\ [\prod_{r=1}^p (1 - (\tilde{\phi}_{gh}^r)^2)^{\tilde{\zeta}_r} - \prod_{r=1}^p (1 - (\tilde{\phi}_{gh}^r)^2 - (\tilde{\rho}_{gh}^r)^2)^{\tilde{\zeta}_r}]^{1/2} \end{array} \right). \tag{35}$$

Step 3. Calculate the average solution for each attribute with Eq. (36):

$$\bar{\tilde{z}}_h = \frac{1}{s} (\oplus_{g=1}^s \tilde{z}_{gh}) = (\bar{\tilde{\omega}}_h, \bar{\tilde{\gamma}}_h, \bar{\tilde{\phi}}_h), \tag{36}$$

where $\bar{\tilde{z}}_h$ denotes the average solution of the h th attribute.

Step 4. Compute the distance from the average solution for each alternative under different attributes by Eq. (37):

$$d(\bar{z}_{gh}, \bar{\bar{z}}_h) = \frac{1}{2} \left(|\bar{\omega}_{gh}^2 - \bar{\bar{\omega}}_h^2| + |\bar{\gamma}_{gh}^2 - \bar{\bar{\gamma}}_h^2| + |\bar{\phi}_{gh}^2 - \bar{\bar{\phi}}_h^2| \right), \tag{37}$$

where $d(\bar{z}_{gh}, \bar{\bar{z}}_h)$ denotes the distance between the g th alternative and the average solution under the h th attribute, and $g = 1, 2, \dots, s; h = 1, 2, \dots, t$.

Step 5. Construct prospect PDA (PPDA) and prospect NDA (PNDA).

For benefit attribute, PPDA and PNDA are given as:

$$PPDA_{gh} = \begin{cases} \frac{\max\left(0, \left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\alpha\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{z}_{gh}) \geq S''(\bar{\bar{z}}_h) \\ \frac{\max\left(0, -\lambda \left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\beta\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{z}_{gh}) < S''(\bar{\bar{z}}_h) \end{cases}, \tag{38}$$

and

$$PNDA_{gh} = \begin{cases} \frac{\max\left(0, \lambda \left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\beta\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{\bar{z}}_h) \geq S''(\bar{z}_{gh}) \\ \frac{\max\left(0, -\left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\alpha\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{\bar{z}}_h) < S''(\bar{z}_{gh}) \end{cases}. \tag{39}$$

Furthermore, PPDA and PNDA of cost attributes depending on Eqs (40)–(41):

$$PPDA_{gh} = \begin{cases} \frac{\max\left(0, \left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\alpha\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{\bar{z}}_h) \geq S''(\bar{z}_{gh}) \\ \frac{\max\left(0, -\lambda \left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\beta\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{\bar{z}}_h) < S''(\bar{z}_{gh}) \end{cases}, \tag{40}$$

and

$$PNDA_{gh} = \begin{cases} \frac{\max\left(0, \lambda \left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\beta\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{z}_{gh}) \geq S''(\bar{\bar{z}}_h) \\ \frac{\max\left(0, -\left(d(\bar{z}_{gh}, \bar{\bar{z}}_h)\right)^\alpha\right)}{S''(\bar{\bar{z}}_h)}, & \text{if } S''(\bar{z}_{gh}) < S''(\bar{\bar{z}}_h) \end{cases}, \tag{41}$$

where α , β and λ are parameters defined in Section 2.4. Usually, $\alpha = 0.88$, $\beta = 0.88$, $\lambda = 2.25$ based on experimental verification of Kahneman and Tversky (1979).

Step 6. Use the MEREC to obtain attribute weight in SFSs, the calculation processes are as follows:

- (1) Construct the normalized spherical fuzzy group score values

$$a_{gh} = \begin{cases} \frac{\min_g S''(\bar{z}_{gh})}{S''(\bar{z}_{gh})}, & \text{for benefit attribute} \\ \frac{S''(\bar{z}_{gh})}{\max_g S''(\bar{z}_{gh})}, & \text{for cost attribute} \end{cases}, \quad (42)$$

where $S''(\bar{z}_{gh})$ denotes score of \bar{z}_{gh} , which is obtained by Eq. (34).

(2) Calculate the overall performance A_g of each alternative.

$$A_g = \ln \left(1 + \left(\frac{1}{t} \sum_{h=1}^t |\ln a_{gh}| \right) \right), \quad g = 1, 2, \dots, s. \quad (43)$$

(3) Figure out the performance of alternatives by removing each attribute

$$\hat{A}_{gh} = \ln \left(1 + \left(\frac{1}{t} \sum_{j=1, j \neq h}^t |\ln a_{gj}| \right) \right), \quad g = 1, 2, \dots, s, h = 1, 2, \dots, t. \quad (44)$$

where \hat{A}_{gh} is the overall performance of alternative ST_g after removing attribute SS_h .

(4) Compute the summation of absolute deviations by Eq. (45):

$$SD_h = \sum_{g=1}^s \left| \hat{A}_{gh} - A_g \right|, \quad h = 1, 2, \dots, t. \quad (45)$$

(5) Get the weight of each attribute by Eq. (46):

$$\tilde{\omega}_h = \frac{SD_h}{\sum_{h=1}^t SD_h}, \quad h = 1, 2, \dots, t, \quad (46)$$

where $\tilde{\omega}_h$ indicates the weight for the h th attribute.

Step 7. Determine probability weights.

$$\tau_{gh} = \begin{cases} \frac{\tilde{\omega}_h^\eta}{(\tilde{\omega}_h^\eta + (1 - \tilde{\omega}_h)^\eta)^\eta}, & \text{if } S''(\bar{z}_{gh}) \geq S''(\bar{z}_h) \\ \frac{\tilde{\omega}_h^\delta}{(\tilde{\omega}_h^\delta + (1 - \tilde{\omega}_h)^\delta)^\delta}, & \text{if } S''(\bar{z}_{gh}) < S''(\bar{z}_h) \end{cases}, \quad (47)$$

where η, δ are parameters defined in Section 2.4. In general, $\eta = 0.61, \delta = 0.69$ based on experimental verification of Tversky and Kahneman (1992).

Step 8. Get the weighted PPDA as well as the weighted PNDA by Eqs (48)–(49):

$$SP_g = \sum_{h=1}^t \tau_{gh} \cdot PPDA_{gh}, \quad g = 1, 2, \dots, s; \quad (48)$$

$$SN_g = \sum_{h=1}^t \tau_{gh} \cdot PNDA_{gh}, \quad g = 1, 2, \dots, s, \tag{49}$$

where SP_g and SN_g represent the weighted PPDA and weighted PNDA of the g th alternative respectively.

Step 9. Standardize the weighted PPDA as well as the weighted PNDA by Eqs (50)–(51):

$$NSP_g = \frac{SP_g}{\max_g SP_g}, \quad g = 1, 2, \dots, s; \tag{50}$$

$$NSN_g = 1 - \frac{SN_g}{\max_g SN_g}, \quad g = 1, 2, \dots, s. \tag{51}$$

Step 10. Calculate the evaluation score ES_g for each alternative with Eq. (52) and get the desirable alternative. Among them, the higher the evaluation score is, the better the alternative will be.

$$ES_g = \frac{1}{2}(NSP_g + NSN_g), \quad g = 1, 2, \dots, s. \tag{52}$$

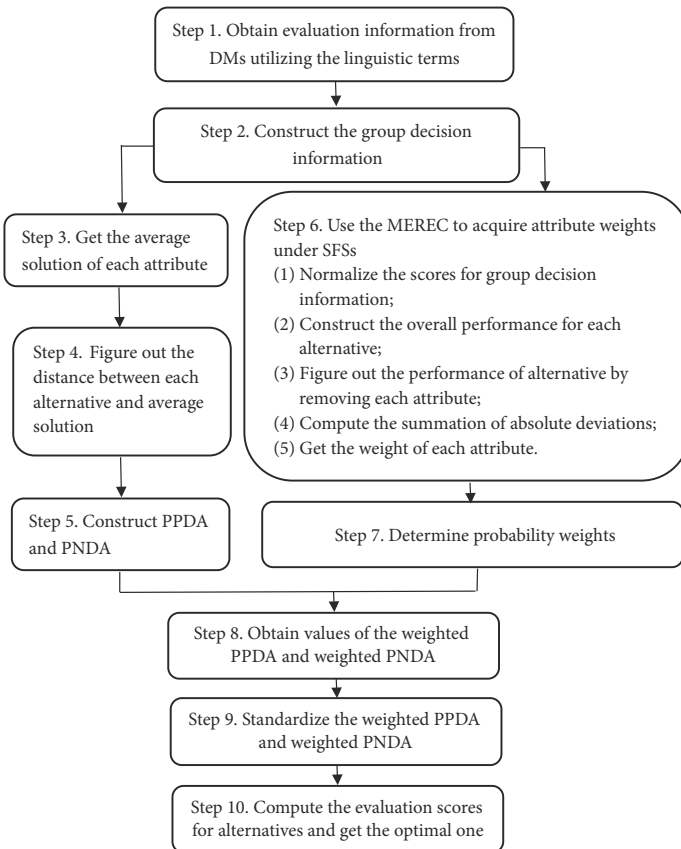


Figure 1. The flow chart for SF-PT-EDAS-MEREC model

Table 1. Linguistic terms and their corresponding SFNs (Gundogdu & Kahraman, 2019)

Linguistic terms	$(\bar{\phi}, \bar{\sigma}, \bar{\rho})$
Extremely very significant (EVS)	(0.9,0.1,0.1)
Very significant (VS)	(0.8,0.2,0.2)
Significant (S)	(0.7,0.3,0.3)
Relative significant (RS)	(0.6,0.4,0.4)
Averagely significant (AS)	(0.5,0.5,0.5)
Slightly significant (SLS)	(0.4,0.6,0.4)
Not significant (NS)	(0.3,0.7,0.3)
Very insignificant (VU)	(0.2,0.8,0.2)
Extremely very insignificant (EVI)	(0.1,0.9,0.1)

4. Numerical examples and comparative analysis

4.1. The numerical example of spherical fuzzy MAGDM

Example 1. The rapid development of economy promotes the continuous increase of people’s income. In today’s society, people’s awareness of financial management and investment is increasing day by day, more and more investors choose to invest in stocks to seek excess returns. However, stock investment is a high risk and high return, blind investment may lead to unsatisfactory investment returns and even bring economic losses. So scientific and reasonable stock selection is of great significance for investors to reduce investment risk and get better investment return. In order to choose the most suitable stocks to invest, some investors invite three experts (EX_1, EX_2, EX_3) to evaluate the five candidate stocks ST_e ($e = 1, \dots, 5$) by considering the following four attributes: SS_1 (Earnings per share), SS_2 (Net value per share), SS_3 (Profit growth rate), SS_4 (Asset-liability ratio). Among them, the others are benefit attributes except SS_4 . Furthermore, $\bar{c} = (0.40, 0.35, 0.25)^T$ denotes experts’ weight vector, and attribute weight information is unknown. The evaluation information from EX_1, EX_2, EX_3 with SFNs are displayed in Table 2. Then we employ SF-PT-EDAS-MEREC model to help investors select the best stock.

Table 2. Evaluation information from experts

DMs	Alternatives	SS_1	SS_2	SS_3	SS_4
EX_1	ST_1	S	VS	SLS	AS
	ST_2	S	SLS	VS	RS
	ST_3	VS	S	EVS	AS
	ST_4	S	NS	RS	S
	ST_5	RS	VU	S	S

End of Table 2

DMs	Alternatives	SS ₁	SS ₂	SS ₃	SS ₄
EX ₂	ST ₁	VU	RS	EVS	VU
	ST ₂	EVS	AS	S	VS
	ST ₃	EVI	S	S	NS
	ST ₄	S	SLS	RS	S
	ST ₅	S	RS	NS	VU
EX ₃	ST ₁	VU	VU	RS	AS
	ST ₂	VS	AS	VS	VS
	ST ₃	NS	SLS	AS	NS
	ST ₄	VS	VU	NS	RS
	ST ₅	RS	VS	VS	AS

Step 1. Assessment information from experts is given in Table 2, then so Step 1 is done.

Step 2. Aggregate individual evaluation information by Eq. (35) to get group decision information (See Table 3).

Table 3. The group decision information

Alternatives	SS ₁	SS ₂	SS ₃	SS ₄
ST ₁	(0.50,0.54,0.27)	(0.66,0.36,0.29)	(0.73,0.29,0.27)	(0.43,0.59,0.45)
ST ₂	(0.82,0.18,0.20)	(0.46,0.54,0.47)	(0.77,0.23,0.23)	(0.74,0.26,0.28)
ST ₃	(0.59,0.46,0.22)	(0.65,0.36,0.32)	(0.79,0.22,0.26)	(0.40,0.61,0.41)
ST ₄	(0.73,0.27,0.27)	(0.32,0.69,0.33)	(0.55,0.46,0.39)	(0.68,0.32,0.32)
ST ₅	(0.64,0.36,0.36)	(0.59,0.44,0.30)	(0.65,0.36,0.28)	(0.55,0.48,0.35)

Step 3. Calculate the average solution of each attribute with (36) as shown in Table 4.

Table 4. The average solution of each attribute

SS ₁	SS ₂	SS ₃	SS ₄
(0.68,0.34,0.27)	(0.56,0.46,0.35)	(0.71,0.30,0.28)	(0.59,0.43,0.36)

Step 4. Compute the distance from the average solution for each alternative under different attributes by Eq. (37) (See Table 5).

Table 5. The distance of each alternative from the average solution under different attributes

Alternatives	SS ₁	SS ₂	SS ₃	SS ₄
ST ₁	0.19329	0.12052	0.01869	0.19880
ST ₂	0.16012	0.13864	0.07410	0.18329
ST ₃	0.11787	0.10354	0.08517	0.20995
ST ₄	0.05593	0.24146	0.19895	0.10848
ST ₅	0.06530	0.03904	0.06376	0.04980

Step 5. Calculate PPDA and PNDA of each attribute by Eqs (38)–(41), as illustrated in Tables 6–7 (where $\alpha = 0.88$, $\beta = 0.88$, $\lambda = 2.25$ based on the study of Kahneman and Tversky (1979), they have been accepted by most researchers).

Table 6. The PPDA for each attribute

Alternatives	SS ₁	SS ₂	SS ₃	SS ₄
ST ₁	0.00000	0.48645	0.06631	0.70559
ST ₂	0.47886	0.00000	0.22289	0.00000
ST ₃	0.00000	0.42560	0.25193	0.74031
ST ₄	0.18977	0.00000	0.00000	0.00000
ST ₅	0.00000	0.18039	0.00000	0.20868

Table 7. The PNDA for each attribute

Alternative	SS ₁	SS ₂	SS ₃	SS ₄
ST ₁	1.27159	0.00000	0.00000	0.00000
ST ₂	0.00000	1.23809	0.00000	1.47805
ST ₃	0.82283	0.00000	0.00000	0.00000
ST ₄	0.00000	2.01743	1.19594	0.93162
ST ₅	0.48930	0.00000	0.43936	0.00000

Step 6. Obtain the attribute weights with Eqs (42)–(46), the calculation processes are shown in Table 8 to Table 12 as follows:

Table 8. Normalized group score values

Alternatives	SS ₁	SS ₂	SS ₃	SS ₄
ST ₁	1.00000	0.44983	0.65421	0.47733
ST ₂	0.46527	0.69998	0.59785	1.00000
ST ₃	0.82094	0.45999	0.56146	0.44361
ST ₄	0.58478	1.00000	1.00000	0.86271
ST ₅	0.71558	0.53146	0.80255	0.63606

Table 9. Overall performance A_g for each alternative

	SS ₁	SS ₂	SS ₃	SS ₄	SS ₁
A_g	0.39924	0.34293	0.46434	0.15790	0.34345

Table 10. The values of \hat{A}_{gh}

Alternatives	SS ₁	SS ₂	SS ₃	SS ₄
ST ₁	0.39924	0.25539	0.32542	0.26682
ST ₂	0.19703	0.27755	0.24722	0.34293
ST ₃	0.43285	0.33421	0.36926	0.32770
ST ₄	0.03625	0.15790	0.15790	0.12586
ST ₅	0.28227	0.22456	0.30367	0.25981

Table 11. The sum of absolute deviations SD_h for each attribute

	SS_1	SS_2	SS_3	SS_4
SD_h	0.36021	0.45825	0.30440	0.38474

Table 12. Attribute weight $\tilde{\omega}_h$

	SS_1	SS_2	SS_3	SS_4
$\tilde{\omega}_h$	0.23893	0.30396	0.20191	0.25520

Step 7. Determine probability weights based on Eq. (47) (where $\eta = 0.61$, $\delta = 0.69$ based on the study of Tversky and Kahneman (1992), they have been accepted by most researchers.) (See Table 13).

Table 13. Probability weights

Alternatives	SS_1	SS_2	SS_3	SS_4
ST_1	0.28569	0.32049	0.26197	0.29716
ST_2	0.28435	0.33020	0.26197	0.29371
ST_3	0.28569	0.32049	0.26197	0.29716
ST_4	0.28435	0.33020	0.25848	0.29371
ST_5	0.28569	0.32049	0.25848	0.29716

Step 8. Determine the weighted PPDA and weighted PNDA for the alternatives by Eqs (48)–(49), as displayed in Table 14.

Table 14. Weighted PPDA and weighted PNDA

Alternatives	Weighted PPDA (SP_g)	Weighted PNDA (SN_g)
ST_1	0.38294	0.36328
ST_2	0.19455	0.84292
ST_3	0.42238	0.23507
ST_4	0.05396	1.24889
ST_5	0.11983	0.25335

Step 9. Standardize the weighted PPDA as well as weighted PNDA with Eqs (50)–(51) (See Table 15).

Table 15. Normalized weighted PPDA and weighted PNDA

Alternatives	Normalized weighted PPDA (SP_g)	Normalized weighted PNDA (SN_g)
ST_1	0.90662	0.70912
ST_2	0.46060	0.32506
ST_3	1.00000	0.81178
ST_4	0.12775	0.00000
ST_5	0.28369	0.79714

Step 10. Determine evaluation score for each alternative by Eq. (52) and rank evaluation scores to get the ideal alternative (See Table 16).

Table 16. Evaluation scores of alternatives and ranking order

Alternatives	Evaluation scores of alternatives (ES_g)	Ranking order
ST_1	0.80787	$ST_3 > ST_1 > ST_5 > ST_2 > S_{T4}$
ST_2	0.39283	
ST_3	0.90589	
ST_4	0.06388	
ST_5	0.54041	

By Table 16, ST_3 is the best alternative.

4.2. Comparative analysis

4.2.1. Compare SF-PT-EDAS-MEREC with some spherical fuzzy operators

In this subsection, we compare the presented SF-PT-EDAS-MEREC model with SWAM operator (Gundogdu & Kahraman, 2019), SWGM operator (Gundogdu & Kahraman, 2019), spherical weighted averaging aggregation (SFNWAA) operator (Ashraf et al., 2019), spherical weighted geometric aggregation (SFNWGA) operator (Ashraf et al., 2019), spherical fuzzy weighted averaging interaction (SFWAI) operator (Ju et al., 2021) as well as spherical fuzzy weighted geometric interaction (SFWGI) operator (Ju et al., 2021). As can be seen from Table 17 to Table 22, ST_3 is always the best alternative.

Table 17. SWAM operator and ranking order

Alternatives	SWAM	Scores	Ranking order
ST_1	(0.64502, 0.38055, 0.31482)	0.10471	$ST_3 > ST_2 > ST_1 > ST_5 > ST_4$
ST_2	(0.64759, 0.37723, 0.31470)	0.10690	
ST_3	(0.67153, 0.34702, 0.30699)	0.13128	
ST_4	(0.52075, 0.50451, 0.33057)	0.00591	
ST_5	(0.62118, 0.39933, 0.31939)	0.08469	

Table 18. SWGM operator and ranking order

Alternatives	SWGM	Scores	Ranking order
ST_1	(0.49128, 0.54572, 0.33054)	-0.02046	$ST_3 > ST_2 > ST_5 > ST_1 > ST_4$
ST_2	(0.50232, 0.53222, 0.34588)	-0.01025	
ST_3	(0.53216, 0.51215, 0.32230)	0.00800	
ST_4	(0.41877, 0.59881, 0.32644)	-0.06566	
ST_5	(0.50240, 0.52937, 0.32179)	-0.01047	

Table 19. SFNWAA operator and ranking order

Alternatives	SFNWAA	Scores	Ranking order
ST_1	(0.64502, 0.38055, 0.27172)	0.66425	$ST_3 > ST_1 > ST_2 > ST_5 > ST_4$
ST_2	(0.64759, 0.37723, 0.27863)	0.66391	
ST_3	(0.67153, 0.34702, 0.26619)	0.68610	
ST_4	(0.52075, 0.50451, 0.31146)	0.56826	
ST_5	(0.62118, 0.39933, 0.29145)	0.64347	

Table 20. SFNWGA operator and ranking order

Alternatives	SFNWGA	Scores	Ranking order
ST_1	(0.49128, 0.54572, 0.27172)	0.55795	$ST_3 > ST_2 > ST_5 > ST_1 > ST_4$
ST_2	(0.50232, 0.53222, 0.27863)	0.56382	
ST_3	(0.53216, 0.51215, 0.26619)	0.58460	
ST_4	(0.41877, 0.59881, 0.31146)	0.50283	
ST_5	(0.50240, 0.52937, 0.29145)	0.56053	

Table 21. SFWAI operator and ranking order

Alternatives	SFWAI	Scores	Ranking order
ST_1	(0.64502, 0.45462, 0.31482)	0.55513	$ST_3 > ST_2 > ST_1 > ST_5 > ST_4$
ST_2	(0.64759, 0.45061, 0.31470)	0.55864	
ST_3	(0.67153, 0.41981, 0.30699)	0.59023	
ST_4	(0.52075, 0.54773, 0.33057)	0.43095	
ST_5	(0.62118, 0.45591, 0.31939)	0.53800	

Table 22. SFWGI operator and ranking order

Alternatives	SFWGI	Scores	Ranking order
ST_1	(0.56106, 0.54572, 0.33054)	0.45386	$ST_3 > ST_5 > ST_2 > ST_1 > ST_4$
ST_2	(0.56442, 0.53222, 0.34588)	0.45784	
ST_3	(0.59603, 0.51215, 0.32230)	0.49454	
ST_4	(0.46402, 0.59881, 0.32644)	0.37509	
ST_5	(0.55853, 0.52937, 0.32179)	0.46408	

4.2.2. Compare SF-PT-EDAS-MEREC model with some existing MAGDM approaches in SFs

This subsection we employ the spherical fuzzy TOPSIS (SF-TOPSIS) approach (Kutlu Gündoğdu & Kahraman, 2021), spherical fuzzy EDAS (SF-EDAS) approach (Menekse & Akdag, 2022) and spherical fuzzy WASPAS (SF-WASPAS) approach (Boltürk & Kutlu Gündoğdu, 2021) to attest the legality for the developed model. In the light of the data in Table 2 as well as attribute weights, the computing results are listed in Table 23 to Table 25 respectively.

Table 23. The closeness ratios for alternatives and ranking order using SF-TOPSIS

	ST_1	ST_2	ST_3	ST_4	ST_5
Closeness ratios	0.57414	0.53635	0.64261	0.31971	0.57476
Ranking order	$ST_3 > ST_5 > ST_1 > ST_2 > ST_4$				

Table 24. The appraisal scores and ranking order of alternatives by SF-EDAS

	ST_1	ST_2	ST_3	ST_4	ST_5
Appraisal scores	0.27060	0.16712	0.69594	0.06760	0.32533
Ranking order	$ST_3 > ST_5 > ST_1 > ST_2 > ST_4$				

Table 25. The joint generalized scores and ranking order of alternatives by SF-WASPAS

	ST_1	ST_2	ST_3	ST_4	ST_5
Joint generalized scores	0.09887	0.05590	0.12572	-0.02128	0.08414
Ranking order	$ST_3 > ST_1 > ST_5 > ST_2 > ST_4$				

Clearly, as you can see from Tables 23–25, alternative ST_3 is always the best. Moreover, Table 26 gives the ranking order for different approaches.

Table 26. The ranging order for different methods

Methods	Ranking orders
SWAM (Gundogdu & Kahraman, 2019)	$ST_3 > ST_2 > ST_1 > ST_5 > ST_4$
SWGGM (Gundogdu & Kahraman, 2019)	$ST_3 > ST_2 > ST_5 > ST_1 > ST_4$
SFNWAA (Ashraf et al., 2019)	$ST_3 > ST_1 > ST_2 > ST_5 > ST_4$
SFNWGA (Ashraf et al., 2019)	$ST_3 > ST_2 > ST_5 > ST_1 > ST_4$
SFWAI (Ju et al., 2021)	$ST_3 > ST_2 > ST_1 > ST_5 > ST_4$
SFWGI (Ju et al., 2021)	$ST_3 > ST_5 > ST_2 > ST_1 > ST_4$
SF-TOPSIS (Kutlu Gündoğdu & Kahraman, 2021)	$ST_3 > ST_5 > ST_1 > ST_2 > ST_4$
SF-EDAS (Menekse & Akdag, 2022)	$ST_3 > ST_5 > ST_1 > ST_2 > ST_4$
SF-WASPAS (Boltürk & Kutlu Gündoğdu, 2021) (threshold parameter $\mu = 0.5$)	$ST_3 > ST_1 > ST_5 > ST_2 > ST_4$
The proposed SF-PT-EDAS-MEREC	$ST_3 > ST_1 > ST_5 > ST_2 > ST_4$

4.2.3. Contrastive analysis

By Table 26, with the exception of SF-WASPAS approach, the ordering of the SF-PT-EDAS-MEREC model is different from that of the other aforementioned existing methods. But the choices for best and worst alternatives are consistent among all methods. The above comparison strongly explains the legality for SF-PT-EDAS-MEREC model. In the decision-making process, SWAM, SWGM, SFNWAA and SFWGA operators emphasize the overall impact, whereas SFWAI as well as SFWGI operators focus on individual effect. SF-TOPSIS approach evaluate each alternative by measuring its distance from the ideal solutions. SF-EDAS ap-

proach obtains the optimal alternative by measuring the positive and negative distances from the average values. SF-WASPAS approach integrates weighted sum model as well as weighted product model in evaluating each set of attributes for different alternatives. Each method has its own characteristics. However, the developed model not only utilizes the simplicity and stability of EDAS method in the process of alternative ranking, but also uses PT to fully consider the DMs' risk attitude when facing gains and losses. Furthermore, we use the developed new score function to extend MEREC to SFSs in objectively obtaining unknown attribute weights. Therefore, the proposed model will be more scientific and practical for stock investment selection.

4.3. Another numerical example of spherical fuzzy MAGDM

Example 2. In this subsection, a new example of stock investment selection (adapted from Zhao et al., 2021) is presented to further illustrate the effectiveness of the proposed model. In the stock market, how to choose valuable stocks efficaciously is the key issue of investment decision. For investors who invest in stocks for the long term, it is important not only to focus on the companies corresponding to the stocks, but also to consider the amount of shareholder returns and the future development prospects of the industry. According to the above analysis and investigation, the following four indicators SS_h ($h = 1, 2, \dots, 4$) are chosen as the evaluation factors for stock investment: (1) SS_1 is the industrial development prospect, (2) SS_2 is the influence degree of economic environment, (3) SS_3 is the sustainable competitiveness of enterprises, (4) SS_4 is the degree of stock market price below its intrinsic value. Among them, SS_2 is the cost attribute. Three senior experts evaluated the five candidate stocks ST_e ($e = 1, \dots, 5$) according to the above four attributes. $\bar{\zeta} = (0.28, 0.42, 0.30)^T$ denotes experts' weight vector, and attribute weight information is unknown. The evaluation information from three senior experts with SFNs are displayed in Tables 27–29.

Table 27. Evaluation information from expert 1 in Example 2

Alternatives	SS_1	SS_2
ST_1	(0.3856,0.5529,0.0615)	(0.2231,0.3628,0.4141)
ST_2	(0.2576,0.3281,0.4143)	(0.3587,0.2945,0.3468)
ST_3	(0.6394,0.2419,0.1187)	(0.5412,0.2198,0.2390)
ST_4	(0.2688,0.6523,0.0789)	(0.6451,0.2548,0.1001)
ST_5	(0.6231,0.3514,0.0255)	(0.3561,0.5482,0.0957)
	SS_3	SS_4
ST_1	(0.4512,0.3264,0.2224)	(0.2287,0.4256,0.3457)
ST_2	(0.2366,0.6809,0.0825)	(0.6201,0.3692,0.0107)
ST_3	(0.7012,0.2654,0.0334)	(0.5480,0.3648,0.0872)
ST_4	(0.3642,0.3974,0.2384)	(0.6946,0.3021,0.0033)
ST_5	(0.3422,0.4593,0.1985)	(0.1024,0.7452,0.1524)

Table 28. Evaluation information from expert 2 in Example 2

Alternatives	SS ₁	SS ₂
ST ₁	(0.4125,0.3298,0.2577)	(0.2654,0.6542,0.0804)
ST ₂	(0.4548,0.2212,0.3240)	(0.5213,0.3505,0.1282)
ST ₃	(0.8121,0.0023,0.1856)	(0.7111,0.2568,0.0321)
ST ₄	(0.3254,0.3649,0.3097)	(0.7032,0.2513,0.0455)
ST ₅	(0.7412,0.1011,0.1577)	(0.3458,0.2649,0.3893)
	SS ₃	SS ₄
ST ₁	(0.1165,0.3648,0.5187)	(0.3454,0.6512,0.0034)
ST ₂	(0.6245,0.3614,0.0141)	(0.3344,0.6215,0.0441)
ST ₃	(0.5807,0.3196,0.0997)	(0.3681,0.4982,0.1337)
ST ₄	(0.2569,0.6154,0.1277)	(0.6412,0.3216,0.0372)
ST ₅	(0.4485,0.3151,0.2364)	(0.6635,0.0257,0.3108)

Table 29. Evaluation information from expert 3 in Example 2

Alternatives	SS ₁	SS ₂
ST ₁	(0.4261,0.2357,0.3382)	(0.6244,0.3011,0.0745)
ST ₂	(0.5214,0.3548,0.1238)	(0.3257,0.3389,0.3354)
ST ₃	(0.7331,0.2199,0.0470)	(0.3518,0.6411,0.0071)
ST ₄	(0.6542,0.1798,0.1660)	(0.3694,0.2584,0.3722)
ST ₅	(0.1241,0.6157,0.2602)	(0.7122,0.1113,0.1765)
	SS ₃	SS ₄
ST ₁	(0.3254,0.1277,0.5469)	(0.3658,0.6222,0.0120)
ST ₂	(0.1213,0.7418,0.1369)	(0.4533,0.2134,0.3333)
ST ₃	(0.5432,0.3588,0.0980)	(0.5423,0.3125,0.1452)
ST ₄	(0.6425,0.3201,0.0374)	(0.6152,0.3648,0.0200)
ST ₅	(0.2548,0.4567,0.2885)	(0.6245,0.3657,0.0098)

Next, we use the proposed model and the aforementioned existing methods to solve Example 2 respectively, and the evaluation results are shown in Table 30.

As can be seen from Table 30, the ranking of SF-PT-EDAS-MEREC method is different from the existing methods, but the optimal scheme selection of all methods is ST₃. The above analysis further proves the effectiveness of our proposed model. In addition, as mentioned in the comparative analysis of subsection 4.2.3, different methods have their own characteristics in the evaluation of schemes. However, compared with the existing methods, the proposed model not only considers the psychological behavior factors of DMs in the evaluation process, but also provides a new research perspective for the reasonable acquisition of attribute weights in fuzzy environment via using the newly proposed score function to extend MEREC to SFSS. In addition, compared with the proposed model in reference (Zhao et al., 2021),

Table 30. The ranking order for different methods in Example 2

Methods	Ranking orders
SWAM (Gundogdu & Kahraman, 2019)	$ST_3 > ST_5 > ST_4 > ST_2 > ST_1$
SWGGM (Gundogdu & Kahraman, 2019)	$ST_3 > ST_1 > ST_5 > ST_2 > ST_4$
SFNWAA (Ashraf et al., 2019)	$ST_3 > ST_5 > ST_4 > ST_1 > ST_2$
SFNWGA (Ashraf et al., 2019)	$ST_3 > ST_4 > ST_2 > ST_1 > ST_5$
SFWAI (Ju et al., 2021)	$ST_3 > ST_5 > ST_4 > ST_2 > ST_1$
SFWGI (Ju et al., 2021)	$ST_3 > ST_5 > ST_4 > ST_2 > ST_1$
SF-TOPSIS (Kutlu Gündoğdu & Kahraman, 2021)	$ST_3 > ST_5 > ST_1 > ST_4 > ST_2$
SF-EDAS (Menekse & Akdag, 2022)	$ST_3 > ST_5 > ST_1 > ST_4 > ST_2$
SF-WASPAS (Boltürk & Kutlu Gündoğdu, 2021) (threshold parameter $\mu = 0.5$)	$ST_3 > ST_5 > ST_4 > ST_2 > ST_1$
The proposed SF-PT-EDAS-MEREC	$ST_3 > ST_5 > ST_2 > ST_4 > ST_1$

although the optimal scheme and the worst scheme selection for the two models are consistent, SF-PT-EDAS-MEREC model not only utilizes SFs as tools to express DMs' information preferences more deeply and comprehensively and make up for the deficiency of IFs, but also adopts a new viewpoint which is to obtain attribute weight information effectively and flexibly by measuring the impact of each attribute's removal effect on the overall performance of the alternative. Therefore, the proposed model in this paper is more practical and scientific for dealing with the problem of stock investment selection.

Conclusions

In this article, we develop a new IDSS called SF-PT-EDAS-MEREC model for settling MAGDM issues. Firstly, we review some basic knowledge of SFs, and describe the basic ideas of MEREC, EDAS method and PT. Then we present a novel score function for comparing SFNs more directly and efficiently. What's more, we develop a SF-PT-EDAS-MEREC model by integrating SFs, PT, EDAS method and MEREC for settling uncertain issues. Finally, we use SF-PT-EDAS-MEREC model for the problem of stock investment selection to prove practicability of the established model. In the meantime, we compare the developed model by existing approaches under SFs to further demonstrate its legitimacy and superiority. Hence, the primary achievements for this article are: (1) to propose a novel score function to compare the sizes of SFNs more effectively; (2) to integrate PT into the decision-making process to ameliorate the conventional EDAS approach to fully capture the psychological feelings of DMs; (3) to extend MEREC to acquire unknown attribute weights reasonably in SFs; (4) to develop a SF-PT-EDAS-MEREC model for MAGDM problems; (5) to elucidate the practicability of the established model by two examples of stock investment selection and to attest the effectiveness and meliority for the proposed model by further comparison. The developed model of the thesis not only provides DMs with a wider space of information expression but also fully considers the mental characteristics of DMs when they are faced with gains and losses. Meantime, the MEREC is employed to acquire unknown attribute

weights and enhance the rationality of weight information. Therefore, the presented model will be more scientific and practical in dealing with MAGDM problems. In view of these, future study will apply the established model to assist companies or individuals deal with some other uncertain problems such as project safety evaluation, supplier selection, tourism management evaluation, residential location choice.

Nonetheless, the proposed model also faces some disadvantages. For one thing, the calculation process is relatively complex. For another, the developed model in this paper only considers the case where the evaluation information is SFN, but actual evaluation may confront various different decision-making environments, so subsequent research will focus on the further expansion of the established model in other decision-making environments (e.g., probabilistic hesitant FSs, linguistic neutrosophic sets, Z-numbers, T-spherical fuzzy soft sets). In addition, the established model only uses MEREC to acquire attribute weights by measuring alternatives performances in the decision matrix without thinking about the judgment from experts. Although it reduces the bias of DMs' subjectivity, it also ignores the valuable judgment of some experienced experts. Therefore, future study will also further pay attention to the combination of MEREC and some subjective weight methods (such as SWARA, KEMIRA, FUCOM and DEMATEL), so as to obtain more sound weight information and enhance the accuracy of decision making.

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