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CHOICE OF LOCATION, GROWTH AND WELFARE WITH UNEQUAL POLLUTION EXPOSURES

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Abstract. We develop an endogenous growth model with human capital accumulation in which firms are polluting and heterogeneous individuals must decide, among other things, where to live. The main idea is that pollution is unequally spread across geographical locations, inducing a trade-off for individuals between environmental quality and leisure. In such economy, we show that a better environmental quality and/or a greater degree of inequality lead individuals to favour cleaner locations which, in turn, boosts long-term growth. Welfare-wise, we find that, in general, individuals prefer a greater level of consumption and leisure but lower growth and environmental quality than those which are possible to achieve. Moreover, we show that the sign of the impact of inequality on environmental quality is likely to be negative.

Keywords: location choice, growth, inequality, welfare, environmental quality.

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Introduction

Beside the decisions on the amount of consumption and leisure they purchase, another key variable that enters in individuals' optimization problem is the geographical location for housing. This is an important decision variable because it affects welfare both directly and indirectly in several ways. For instance, housing location, via the distance to travel to

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the working place and the time it requires for its purpose, determines the amount of leisure individuals must give up in addition to their working time. The distance between housing location and working place also involves monetary costs: Glaeser *et al.* (2008), for instance, clearly establish that the cost of automobiles is a relevant factor explaining why poor people often live in the vicinity of the place where they work, while richer individuals choose to live further away. Last, but not least, as many epidemiological studies suggest, location choices determine the pollution burden individuals face. For instance, as emphasized by O'Neill *et al.* (2003), pollution exposure and vulnerability are unequally spread across individuals and mainly depend on their geographical location. This analysis shows in particular that poor individuals tend to live more often in most polluted areas, a feature corroborated by several articles discussing the link between location of households, income and pollution exposure (Michaels, Smith 1990; Kohlhase 1991; Kiel, McClain 1995; McCluskey, Rausser 2003; Kohlhuber *et al.* 2006; Levy 2009; Su *et al.* 2011): this literature shows indeed that poorer individuals tend to live closer to their working place (often characterized by a higher burden of pollution emissions) because it allows them to reduce their cost of commuting to work and housing expenses[1].

In this context and in light of the well recognized and documented effects of pollution on individuals' health, we can infer that such pattern in individuals' geographical location can have important economy-wide consequences, specifically for the determination and interplay of economic variables such as the level of long-term growth and individuals' welfare. As argued by Aloi and Tournemaine (2011, 2013) among others, pollution is a serious and growing problem, particularly in rapidly expanding cities, causing a considerable threat to human health. The problem is that poor health produces significant economic losses not only because it affects individuals' participation to the labour market, but also because it affects individuals' learning abilities. The reason is that health is an important component of human capital which itself is a key engine of long-term growth (Lucas 1988). In other words, capturing the above features in a simple theoretical framework and analyzing the location choices of individuals together with environmental problems in an endogenous growth model is a relevant issue.

In this paper, we explore the impact of inequality and environmental quality on individuals' location choices and determine how it translates to long-term growth, welfare and the relationship between the two. In comparison to existing literature, this article brings a different theoretical perspective as it raises the issue of whether equity, growth and welfare can be mutually compatible, in a context where pollution exposure is uneven and growth and location choices are both endogenous. In the standard theoretical environmental literature,

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1 It is important to mention that we will formalize the centre of economic activity as the place where polluting activities (firms) are located. We should keep in mind, however, that the assertion that poorer households live near the central business district is not always exact. Interested readers could for instance refer to Brueckner *et al.* (1999) among others who have shown that the relative location of individuals depends on the cities' spatial pattern of amenities (a result which, as seen above, is confirmed empirically by Glaeser *et al.* 2008). The pattern of location with respect to pollution, however, which is at the centre of our analysis, seems to be a more common observation. Moreover, as we will see shortly, our simplifying formalization will allow us to capture the fact that, as they become richer, individuals are willing to pay higher transportation costs and housing prices to benefit from a better environmental quality.

Gradus and Smulders (1993), van Ewijk and van Wijnbergen (1995) and Pautrel (2008, 2009), among others, have demonstrated how the negative impact of pollution on learning abilities of individuals can be transmitted to economic growth and shown that a better environmental quality and a higher long-term level of growth are mutually compatible. However, in their models, agents are assumed identical in all dimensions. Moreover, they did not take into account either that pollution is unequally spread across geographical locations, nor the idea that housing location is a decision variable.

In this paper, in contrast, we follow Aloi and Tournemaine (2013) as we develop an endogenous growth model "a la Lucas (1988)" in which heterogeneous individuals must decide not only their level of consumption and the amount of resources to invest in human capital accumulation, but also the place to live (i.e. the distance to commute to work) and their labour supply.

Following Blanchard (2004), heterogeneity across individuals stems from the marginal disutility individuals obtain from non leisure activities, i.e. the time spent at work and that used for commuting to work. The key idea of the model is to formalize a trade-off between environmental quality which affects individuals' learning abilities, and leisure. Specifically, when individuals choose to live closer to the firm, they suffer greater health shortfalls and accumulate less human capital due to the greater impact of pollution coming from production; but on the other hand, they obtain more leisure time as they have a shorter distance to travel to work[2]. We then emphasize the role of environmental quality as a determinant of individuals' location choices, both serving as possible factors affecting their learning abilities, and in turn the level of long-term growth and their welfare.

Close to our analysis, are also the works by Eriksson and Persson (2003) and Kempf and Rossignol (2007) who develop models with heterogeneous individuals. Eriksson and Persson also assume that pollution is unevenly spread across individuals. They study the effect of heterogeneity in income and pollution, together with the society's level of democratization, on environmental policy choices and show that a more even income distribution and more democracy lead to improvements in environmental quality. Kempf and Rossignol (2007) use an AK model with a pollution externality. Their main result is to show that, in general, poorer individuals favour less stringent environmental policies. However, they ignore that pollution exposure and vulnerability disproportionally affect poorer individuals. However, contrary to ours, their model does not analyze how reducing pollution influences growth through the channel of human capital accumulation. This is an important difference since, as emphasized before, the effects of pollution on health and learning abilities represent one of the largest gains from environmental regulation.

Our main results can be summarized as follows. First, we show that a tighter environmental policy always increases individuals' distance of commuting to work. The intuition behind this result is simple. As environmental quality increases, individuals accumulate a greater amount of human capital synonymous of a greater productivity. As a result, they obtain a greater income and become more willing to reduce their amount of leisure to

2 We will see that leisure time increases because individuals reduce their labour supply when they live closer to the firm, source of pollution. The reason is that, in choosing a location near the firm, where pollution is high, individuals accumulate less human capital, i.e. they have a lower productivity.

enjoy a better environmental quality. Second, in contrast with Gradus and Smulders (1993), van Ewijk and van Wijnbergen (1995) and Pautrel (2008, 2009) who find a monotonic relationship between growth and the policy level, we obtain an inverted-U relationship implying the existence of a growth-maximizing policy level. The rationale behind our result is similar to that described in Barro (1990) on the contribution of public services to growth and welfare. In our set up, the environmental policy tool is similar to a fund raising vehicle for abatement investments. Thereby, abatements play a comparable role to public infrastructures, in that the growth-maximizing (abatement) policy reflects two aspects. On the one hand, it reflects the contribution of abatements to the reduction of pollution emissions, which improves the productivity of individuals through their human capital accumulation process. On the other hand, it reflects a resource withdrawal effect, as more resources devoted to abatements have a negative effect on individuals' private investments in the human capital sector.

From a welfare point of view, however, although the theory predicts an ambiguous outcome, the economic intuition and numerical calibration of the model show that, in general, individuals are likely to favour an abatement policy level which is lower than the growth-maximizing one. In other words, the model predicts that, in the most plausible scenario, a greater amount of funds allocated to abatement activities is not only environment and welfare improving but also growth enhancing. Moreover, as we show that the welfare maximizing abatement policy depends on the degree of heterogeneity across individuals (inequality), we can give a simple explanation to the empirical observation according to which an increase in inequality seems to be positively correlated with a reduction in environmental quality (Torras, Boyce 1998; Magnani 2000): formally, we show that a greater degree of inequality across individuals can lead to a reduction of the welfare maximizing abatement policy, synonymous of a greater level of consumption and leisure.

The remainder of the paper is structured as follows. We introduce the model in Section 1. In Section 2, we characterize the equilibrium in which we analyze the abatement policy implications on individuals' location choices, growth and welfare. We finally provide the conclusions of the analysis.

1. Model

The main building block of the model is taken from Aloi and Tournemaine (2013). Consider a closed economy in continuous time populated by a mass $\left[0,N\right]$ of infinitely-lived individuals who live in a city represented by a segment of exogenous length, $\left[0,\alpha^{\max}\right]$. Each individual is endowed with $h_{i,0}=1$ unit of human capital at date zero and must decide the amount of resources to allocate between private consumption and human capital accumulation, the amount of time to work and also the place to live in the city, i.e. the distance to travel to go to work.

Production takes place in a representative firm which, at each instant, produces an output, Y_t, which causes pollution emissions that can be reduced through abatement activities, D_t. To capture the idea that pollution is unequally spread across locations and, possibly, across the population, we assume that the firm producing output is situated on the left hand side of the segment and that, as a result, pollution is more significant in the vicinity of the firm and diminishes as individuals go further away from the firm (see below).

Heterogeneity between individuals stems from their preferences for leisure, or more precisely as we formalize it, from their marginal disutility of work and commuting to work. As we will see below, this is sufficient to introduce income inequality across individuals. The intuition is the following. Individuals who have greater preferences for leisure allocate a lower amount of time to working activities. Therefore, they also have less funds for schooling activities implying that they accumulate less human capital. In addition, we will see that individuals who have greater preferences for leisure will choose to live closer to the firm. The reason is that travelling to the working place can be considered as taking leisure-time away from individuals. As explained in Introduction, this behaviour is in line with substantial evidence indicating that individuals face various trade-offs concerning their choice of location in a city[3].

In the present paper, we incorporate this feature in a stylized way by assuming that commuting costs are welfare reducing. That is, we do not formalize any pecuniary transportation cost. As explained above, our assumption can be rationalized by the fact that commuting costs are time intensive and, thus, might reduce the amount of leisure of an individual.

As we will see, we formalize a trade-off between the time required to commute to the firm where production takes place and environmental quality. Thereby, we endogenize the choice of location of individuals: choosing to live in the vicinity of the firm reduces individuals' welfare cost of commuting, but increases the pollution burden they face, and vice versa. Moreover, introducing heterogeneity in commuting costs will have important implications for the choice of location of individuals and the resulting relationship between growth and inequality. The details of the technologies and preferences are given below.

The technology of output is given by:

$$Y_t = A\int_0^N l_{i,t} h_{i,t} di, \tag{1}$$

where: $A > 0$ is a constant productivity parameter; $l_{i,t}$ is the amount of working-time devoted to output production by individual i; and $h_{i,t}$ is her human capital, where $i \in [0, N]$.[4]

We assume that abatements are public activities, though it would be equivalent to consider that these were private activities. Our approach can be rationalized by appealing to the fact that governments may actually promote the adoption of technologies that reduce pollution originating from the use of resources – such as coal or fuel – impairing air quality. For example, they may promote: "green" buses for public transport, or "green" power stations for energy. Abatements are financed through a flat tax rate τ levied on output production: $D_t = \tau Y_t$. Moreover, we focus on the immediate effects of emissions, such as air pollution, whose implications on health are for the most part direct and are drastically reduced when addressed (Kunzli 2002). Accordingly, we treat pollution as a flow. To account for the idea that

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3 See, for instance, Kim *et al.* (2005) for a more detailed discussion about the potential trade-offs individuals face about their choice of location (in particular between transport, access, space and other attributes), and for their empirical evaluation.

4 In Appendix C, we use a generalized (CES) technology and show that the main results of the paper still hold. Technology (1) has the advantage to simplify the analysis and interpretation of the results. In the same spirit, adding physical capital would complicate, but not "wash away", the effects we are discussing here.

individuals face different levels of pollution depending on their location, we follow Eriksson and Persson (2003) and set the pollution flow faced by individual i as:

$$P_{i,t} = \frac{\left(Y_t / D_t\right)^{\omega}}{\alpha_{i,t}} = \frac{(\tau)^{-\omega}}{\alpha_{i,t}}, \tag{2}$$

where: $\omega > 0$ and $\alpha_{i,t}$ is the choice of location of individual i, i.e. the distance of commuting to work.

As argued in Introduction, the main consequence of pollution is the deterioration of individuals' health and learning abilities. To capture this feature, we set the technology of human capital as:

$$\dot{h}_{i,t} = \frac{\phi\left(\varepsilon_{i,t} Y_{i,t}\right)^{\varphi}\left(\bar{h}_t\right)^{1-\varphi}}{P_{i,t}}, \tag{3}$$

where: $\phi > 0$ is a time-invariant productivity parameter; $0 < 1-\varphi < 1$ is the weight of existing human capital relative to material resources (or, degree of spill-over effect); $\varepsilon_{i,t} Y_{i,t}$ denotes investment in education of individual i, with $\varepsilon_{i,t}$ representing the share of income, $Y_{i,t}$, of individual i and $\bar{h}_t$ is the average level of human capital in the economy. Two comments are in order here. First, the introduction of spill-over effects in the form of average skills in the technology of production of human capital is common practice in the growth literature. The reason is that such spill-over effects are shown to be crucial for human capital convergence. See, for instance, the work of Tamura (1991) and, for empirical support, Alonso-Carrera (2001). This property will be useful when we will turn to the characterization of the steady state. Second, the way pollution affects learning abilities and as a result the pace of human capital accumulation of an individual depends on her choice of location, $\alpha_{i,t}$.

Turning to the specification of preferences, it is assumed that individual i derives utility from her level of consumption, $c_{i,t}$, leisure and environmental quality. Her preferences are represented by:

$$U_{i,t} = \int_0^{\infty} \left\{ \ln c_{i,t} - \beta_i \left[l_{i,t} + \frac{\left(\alpha_{i,t}\right)^{\eta}}{\eta} \right] - \psi \ln P_{i,t} \right\} e^{-\rho t} dt, \tag{4}$$

where: $\eta > 1$, $\rho > 0$ is the rate of time preference; $\beta_i > 0$ is the marginal disutility of work and travelling to work; and $\psi > 0$ measures the weight attached to the environment[5]. This specification allows the income and substitution effects of a change in the real wage to cancel out and for steady-state growth to exhibit constant hours of work and travelling to work. As it is standard in this kind of framework, we assume that $b_i = \ln\beta_i$ is normally distributed with mean $\bar{b}$ and variance σ_b^2, so that β_i is itself log-normally distributed. As mentioned, the

[5] Although assuming a linear marginal disutility of work is not essential for the results we derive in this paper, assuming a non linear disutility of travel to work is necessary to obtain an interior solution. Let us mention that it would be equivalent to conduct the analysis with a utility of the form: $U_{i,t} = \int_0^{\infty} \left\{ \ln c_{i,t} - \beta_i \left[\left(l_{i,t}\right)^{\vartheta} / \vartheta + \left(\alpha_{i,t}\right)^{\eta} / \eta \right] - \psi \ln P_{i,t} \right\} e^{-\rho t} dt$, with $\vartheta \geq 1$. We do not do it to simplify the analysis.

marginal disutility of work and commuting to work, β_i, determines how hard an individual works compared with others. Thus, it can be interpreted as a level of motivation of an individual. Finally, to simplify the analysis, throughout the paper, we assume that the following parameter restriction is verified: $\varphi + 1/\eta < 1$. This will allow us to ensure that a solution exists and is unique in steady state.

2. Equilibrium

In this section, we set out the maximization problem of individuals and analyze the steady-state properties of the model. First, we investigate the long-run effects of heterogeneity in preferences for leisure on labour supply, the choice of location of individuals and growth. Next, we discuss the impacts of abatement policy on these variables. Finally, we analyze the relationship between abatement policy and welfare.

2.1. Efficiency conditions

We assume that the markets of output and human capital are perfectly competitive, and use output as the numeraire. Denoting by $w_{i,t}$ the wage rate of individual i, $i \in [0, N]$, it follows that the competitive firm in the output sector maximizes $\pi_{Yt} = (1-\tau)A\int_0^N l_{i,t}h_{i,t}d_i - \int_0^N w_{i,t}l_{i,t}h_{i,t}d_i$.
Accordingly, the real wage for any individual i, $i \in [0, N]$, is given by:

$$w = (1-\tau)A \text{, for all } i \in [0, N], \tag{5}$$

where throughout the paper, we drop the time index for constant variables.

On the consumer side, each individual i takes as given the level of pollution he/she faces, and chooses consumption, $c_{i,t}$, the fraction of income devoted to education, $\varepsilon_{i,t}$, the path for human capital, $h_{i,t}$, and his/her location $\alpha_{i,t}$ that maximize lifetime utility (4) subject to the law of motion of human capital (3) and the budget constraint given by:

$$c_{i,t} = (1-\varepsilon_{i,t})wl_{i,t}h_{i,t}. \tag{6}$$

After straightforward substitutions, the Current-Value Hamiltonian to this problem is:

$$CVH_{i,t} = \ln\left[(1-\varepsilon_{i,t})(1-\tau)Al_{i,t}h_{i,t}\right] - \beta_i\left[l_{i,t} + \frac{(\alpha_{i,t})^{\eta}}{\eta}\right] + \psi\ln(\alpha_{i,t}) +$$
$$\omega\psi\ln(\tau) + \mu_{i,t}\phi\alpha_{i,t}(\tau)^{\omega}\left[\varepsilon_{i,t}A(1-\tau)l_{i,t}h_{i,t}\right]^{\varphi}\left(\bar{h}_t\right)^{1-\varphi},$$

where $\mu_{i,t}$ is the co-state variable associated to the human capital accumulation process and the abatement policy level is taken as given. The solution to this problem is defined by the first order conditions: $\partial CVH_{i,t}/\partial l_{i,t} = 0$, $\partial CVH_{i,t}/\partial \varepsilon_{i,t} = 0$, $\partial CVH_{i,t}/\partial \alpha_{i,t} = 0$, $\partial CVH_{i,t}/\partial h_{i,t} = 1/h_{i,t} + \mu_{i,t}\varphi\dot{h}_{i,t}/h_{i,t} = -\dot{\mu}_{i,t} + \rho\mu_{i,t}$, along with the transversality condition given by: $\lim_{t\to\infty} \mu_{i,t}h_{i,t}e^{-\rho t} = 0$.

Manipulations of these conditions yield:

$$\beta_i = \frac{1}{l_{i,t}} + \mu_{i,t}\varphi \frac{\dot{h}_{i,t}}{l_{i,t}};$$

$$\frac{1}{\left(1-\varepsilon_{i,t}\right)} = \frac{\mu_{i,t}\varphi \dot{h}_{i,t}}{\varepsilon_{i,t}};$$

$$\beta_i \left(\alpha_{i,t}\right)^{\eta} = \psi + \mu_{i,t}\dot{h}_{i,t};$$

$$\frac{1}{\mu_{i,t} h_{i,t}} + \frac{\varphi \dot{h}_{i,t}}{h_{i,t}} + \frac{\dot{\mu}_{i,t}}{\mu_{i,t}} = \rho .$$

The first expression above states that the marginal utility loss of an extra unit of time spent in output production equals the marginal gain in terms of additional units of output and human capital produced; the expression immediately below states that the marginal (utility) loss in consumption of an extra unit of income allocated to education equals the marginal gain in terms of additional units of human capital produced; the third expression states that the marginal utility loss of an extra unit of time spent to go to work equals the marginal gain in terms of additional units of human capital produced and in terms of welfare gain from a better environment; finally, the last expression states that the return to education equals the discount rate, ρ.

2.2. Steady-state properties

2.2.1. Characterization

Having set out the optimization of each individual, we now characterize the steady state, i.e. we determine the individual labour supply, the share of income devoted to education, the choice of location, and the growth rate of individuals' human capital, income and consumption[6]. To proceed, first we note that, at steady state, the share of income devoted to education and the growth rate of any variable are constant over time. Moreover, growth rates must be the same across individuals, i.e. $g_i = g$ for all $i \in [0, N]$. This property comes from the presence of human capital spill-over, $\bar{h}_t$, in the technology of human capital accumulation (Eq. (3)) which implies that, as the level of human capital in education forges ahead of the average, its growth rate slows down and convergence of human capital growth rates occurs. Using this information, we can express the steady-state labour supply, the share of income devoted to education and the choice of location as follows:

$$l_i = \frac{1}{\beta_i} \frac{g+\rho}{\left(1-\varphi\right)g+\rho}; \tag{7}$$

$$\varepsilon = \frac{\varphi g}{\rho + g}; \tag{8}$$

[6] The analysis of the transitional dynamics is relegated in Appendix B.

and

$$\alpha_i = \left[\frac{g}{\beta_i \left[(1-\varphi) g + \rho \right]} + \frac{\psi}{\beta_i} \right]^{\frac{1}{\eta}} .^{7} \tag{9}$$

From Eq. (3) and the previous results, the common growth rate of human capital, income and consumption is determined by:

$$(g)^{1-\varphi} \left[(1-\varphi) g + \rho \right]^{\varphi} = \phi \left(\frac{1}{\beta_i} \right)^{\frac{1}{\eta}+\varphi} \left[\frac{g}{(1-\varphi) g + \rho} + \psi \right]^{1/\eta} (\varphi A)^{\varphi} (1-\tau)^{\varphi} (\tau)^{\omega} \left(\frac{\overline{h}_t}{h_{i,t}} \right)^{1-\varphi} . \tag{10}$$

Remark that, the labour supply of individuals is negatively correlated with the marginal disutility of work and travelling to work, β_i. This is the ingredient to generate income inequalities across individuals: individuals who are working more generate a greater level of income which allows them to have a greater level of investments in education, and thus a greater level of human capital. We note indeed that, although equality of the growth rates across individuals implies that the share of income allocated to education is the same, in terms of levels, the amount allocated to education differs across individuals due to their different incomes. Finally, the choice of location of individual i is negatively correlated with the marginal disutility of work and commuting to work, β_i. It implies that the more motivated individuals who work harder choose to live further away from the firm. It is important to point out that this property fits with the recent empirical analysis by Gutierrez-i-Puigarnau and van Ommeren (2010). The authors have indeed found a positive relation between the commuting distance to work and weekly labour supply. Among the reasons given to explain this result, the authors argue that, individuals may choose to increase their number of hours worked per day, but simultaneously, reduce their number of workdays. They also mention the idea that, in congested areas, individuals may choose to leave earlier from home or leave later from their workplace, in order to avoid peak hours. In this case also, it may have a positive effect on their labour supply. Note that in light of our model, we can argue that, with such pattern in behaviour, individuals benefit in turn from a better environmental quality. It implies that they accumulate more human capital which boosts long-term growth.

2.2.2. Effects of heterogeneity on labour supply, choice of location and growth

In this section, we analyze the effects of heterogeneity formalized through differences in the marginal disutility of work and travelling to work, β_i. To proceed, we develop a system of the growth rate (g), mean time devoted to work ($\overline{l}$) and average location, ($\overline{\alpha}$). Let us mention that the system developed here has the same structure as the one we would obtain with homogeneous agents. The difference between the two systems comes from the presence of an additional term under heterogeneity which is captured by the variance term, σ_b^2, which

[7] It is implicitly that assumed that $\alpha^{\max}$ is large enough so that $\alpha_i \in \left[0, \alpha^{\max}\right]$ for any $i \in [0, N]$. The case where $\alpha_j > \alpha^{max}$ for some $j \in [0, N]$, is a corner solution in which a sub-set of individuals chooses to live at the limit of the city. Though interesting, this situation is left for future research.

turns out to vanish under symmetry. Thus, the impact of heterogeneity on the determination of economic variables' average can be determined in a straightforward manner.

As shown in Appendix A, manipulation of Eqs. (7)–(10) yields Proposition 1 which summarizes the results we obtain in steady state:

Proposition 1: Under the assumption that $b_i = \ln\beta_i$ is normally distributed with mean $\bar{b}$ and variance σ_b^2 and under the parameter restriction, $\varphi + 1/\eta < 1$ there exists a unique steady-state equilibrium characterized by a constant mean of hours worked, mean location and growth rate given by:[8]

$$\bar{l} = \frac{g+\rho}{(1-\varphi)g+\rho}\exp\left[-\bar{b}+\frac{\sigma_b^2}{2}\right]; \tag{11}$$

$$\bar{\alpha} = \left[\frac{g}{\left[(1-\varphi)g+\rho\right]}+\psi\right]^{\frac{1}{\eta}}\exp\left[-\frac{\bar{b}}{\eta}+\frac{1}{\eta^2}\frac{\sigma_b^2}{2}\right] \tag{12}$$

and

$$(g)^{1-\varphi}\left[(1-\varphi)g+\rho\right]^{\varphi}\times\left[\frac{g}{(1-\varphi)g+\rho}+\psi\right]^{-1/\eta} = \phi(\varphi A)^{\varphi}(\tau)^{\omega}(1-\tau)^{\varphi}\times\exp\left[-\left(\frac{1}{\eta}+\varphi\right)\bar{b}+\frac{\left(\frac{1}{\eta}+\varphi\right)^2}{(1-\varphi)}\frac{\sigma_b^2}{2}\right]. \tag{13}$$

Proof: See Appendix A.

Proposition 1 shows that the mean of hours worked, the mean location and the common growth rate are positively related to the variance σ_b^2. To illustrate the results and provide an order of magnitude of the change in variables resulted by an increase in heterogeneity, we proceed to a numerical simulation of the model. We should keep in mind, however, that such exercise can only provide a rough assessment of the main effects at work, in particular because, as mentioned by Oueslati (2002) among others, we lack strong empirical evidence with respect to the pollution function (Eq. (2)) and the preferences for environmental quality (Eq. (4)). In this context, to calibrate the model, we mainly use benchmark parameter values borrowed from Oueslati (2002), Pautrel (2008, 2009) and Tournemaine and Luangaram (2012): as observed from real world data, we set the share of resources to abatement technologies around 2 percent and choose other parameter values to obtain a plausible level of long-term growth around 2 percent. Table 1 summarizes the benchmark value of parameters and Fig. 1 gives a graphical representation of the comparative static exercises when the degree of heterogeneity across individuals increases by 10 percent (i.e. from $\sigma_b^2 = 0.6$ to $\sigma_b^2 = 0.66$).

[8] The parameter restriction $\varphi + 1/\eta < 1$ is assumed to be verified throughout the paper to ensure the existence of a steady-state solution, in particular in the case $\psi \approx 0$, i.e. when pollution is not an argument of the utility function (Eq. (4)).

Table 1. Benchmark parameter values

Description	Parameter	Benchmark value
Productivity in output production	A	0.05
Elasticity of environmental quality	ω	0.1
Productivity in human capital accumulation	ϕ	0.3
Weight of investment in education	φ	0.6
Elasticity of location choice	η	3
Rate of time preference	ρ	0.05
Preference weight for environment	ψ	0.01
Average marginal disutility of work and commuting	$\bar{b}$	0
Variance marginal disutility of work and commuting	σ_b^2	0.6
Abatement policy	τ	0.02

The relationships depicted in Fig. 1 are well documented in the literature which provides in that sense empirical support to our results. On the link between growth and hours worked and inequality, we can refer to the empirical works by Li and Zou (1998), Barro (1999) and Forbes (2000). In the present paper, the explanation to obtain a positive correlation between these variables is following. Due to the structure of the model, a higher degree of heterogeneity across individuals, σ_b^2, leads to an increase of average labour supply, $\bar{l}$. It then results an increase of average income and in turn of the amount of resources allocated to human capital accumulation boosting the long-run level of growth, g. Interestingly, these effects are accompanied by a greater average distance of commuting to work. Thus, we can summarize our results as follows:

Corollary: *A greater heterogeneity across individuals leads to a greater level of growth, a greater amount of hours worked, and a more scattered population.*

Empirical works by Crenshaw and Ameen (1993) and Sylwester (2003) also find a negative relation between inequality and population density. While the authors do not explicitly consider the reasons behind this observation, they explain that the increase of mobility of individuals could be a factor leading to such result. The reason is that it implies the possibility to move to places where wages are higher meaning that high density population goes along with a more equal society. In this paper, in addition to capture such effect and to give an intuition, we provide another perspective as we emphasize the role played by unequal pollution exposures. We show indeed that as individuals become richer, as a result of a greater degree of heterogeneity, they are willing to incur a greater welfare travel cost: in that sense, individuals trade leisure time for less pollution, giving them the opportunity to accumulate more human capital.

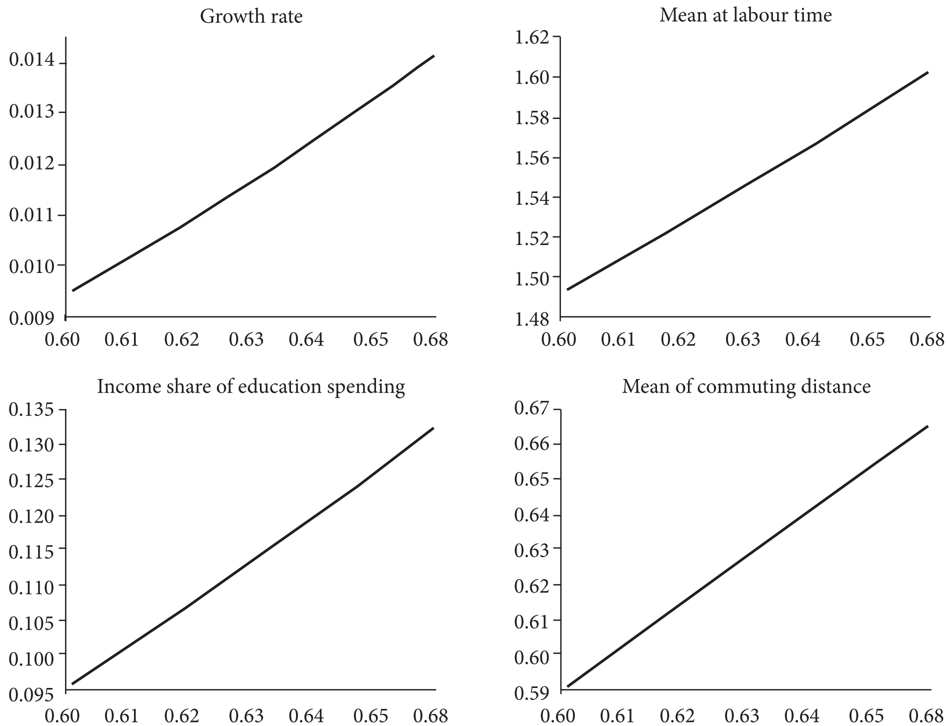

Fig. 1. Effects of heterogeneity on economic variables

2.2.3. Effects of abatement policy on labour supply, choice of location and growth

In this section, we analyze the effects of a change in the level of abatement policy, τ , on the steady-state average labour supply, average location and growth. Using Eqs. (11), (12) and (13), we can establish Proposition 2 whose results are illustrated by Fig. 2 for which we made use of the benchmark parameter values given in Table 1 assuming that the abatement policy verifies: $\tau \in \left(0;0.3\right)$.

Proposition 2: Under the assumption that $b_i = \ln\beta_i$ is normally distributed with mean $\overline{b}$ and variance σ_b^2 and under the parameter restriction, $\varphi + 1/\eta < 1$, the steady-state average labour supply, location and growth rate describe an inverted U-shaped relation with the level of abatement policy, τ . For each variable, the peak is determined at a level, denoted $\tau^{\max}$, verifying the following condition,

$$\tau^{\max} = \frac{\omega}{\omega + \varphi} .^{9}$$

Proof: See Appendix A.

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[9] The baseline parameter values given in Table 1 imply that $\tau^{\max} = 0.143$.

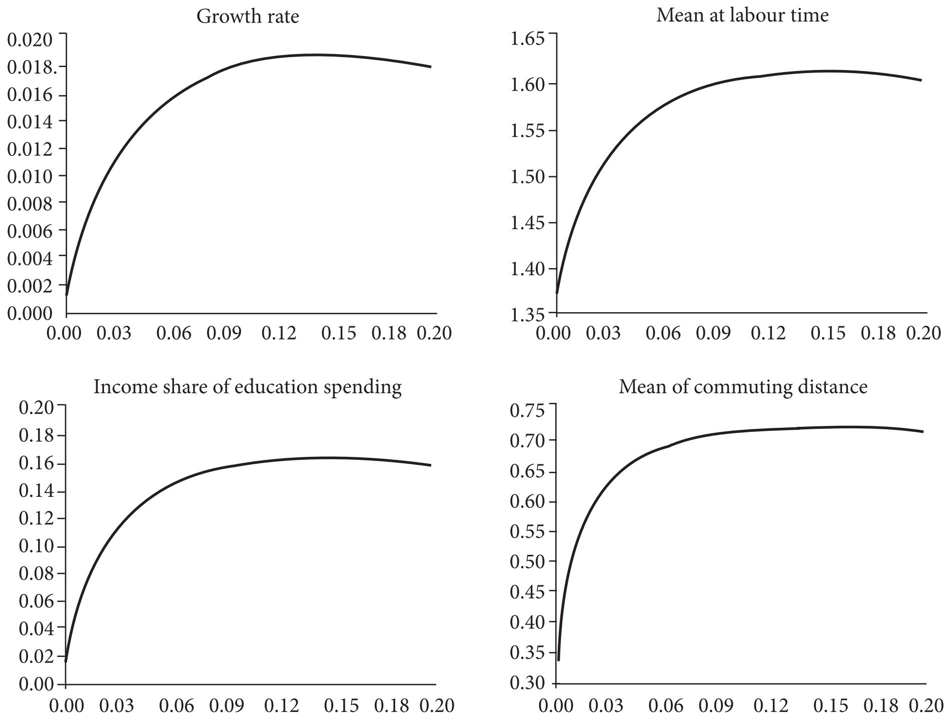

Fig. 2. Effects of the environment policy on economic variables

The inverted-U relationship between abatement policy level and each average variable comes from two effects working in opposite directions. On the one hand, a more stringent abatement policy, synonymous of a better environmental quality, leads to an improvement of the productivity of resources devoted to human capital accumulation which has a positive effect on growth, and also to greater welfare gains as individuals move further away from the firm. On the other hand, a more stringent abatement policy is also taking resources away that could be directly invested in human capital accumulation, via ε, or consumed, i.e. in this latter case it is welfare reducing. Thus, in this case, a tighter abatement policy induces individuals to reduce their investments in education, which is growth reducing and has a negative impact on labour supply and distance between housing location and working place.

Although the opposite effects described above relating growth and abatement policy are known in the literature, they are usually analyzed separately and, as a result, conflicting conclusions are reached regarding the contribution of improved environmental quality to growth. Our work, though, in addition of introducing heterogeneity and considering geographical location as a choice variable, shows that whether a trade-off exists or not between abatement policy and growth depends on the initial level of abatement policy. Formally, we have established that if the level of abatement policy is low (i.e. $0 < \tau < \tau^{\max}$), people work harder, accumulate human capital at a greater pace and move further away from the firm. I.e. as the environmental quality improves, individuals invest more resources in education, are willing to work harder and to support greater travelling costs. In contrast, if the abatement policy is tighter (i.e. $\tau^{max} < \tau < 1$), individuals prefer to work less, live closer to the firm and accumulate human capital at a lower pace. By reducing their labour supply and their travelling burden, individuals compensate, to some extent, the negative impact of a tighter abatement policy.

2.3. Environmental policy choice and welfare

In this section, we characterize the welfare maximizing abatement policy and analyze how this latter is affected by heterogeneity. To proceed, we must compute the level of lifetime utility of individual i. For simplicity, we assume that the economy is initially (i.e. at date zero) in steady state. Hence, the lifetime utility can be expressed in terms of the (exogenously given) initial endowment of human capital, $h_{i,0}$:

$$U_{i,t} = \frac{1}{\rho}\left\{ \begin{array}{c} \ln\left[\frac{(1-\tau)(\tau)^{\omega\psi} Ah_{i,0}}{\beta_i}\right] + \frac{\psi}{\eta}\ln\left[\frac{g}{\beta_i\left[(1-\varphi)g+\rho\right]} + \frac{\psi}{\beta_i}\right] \\ -\frac{g+\rho}{(1-\varphi)g+\rho} - \frac{g}{\eta\left[(1-\varphi)g+\rho\right]} - \frac{\psi}{\eta} \end{array} \right\} + \frac{1}{(\rho)^2} g. \tag{14}$$

Differentiating (14) with respect to τ, we obtain:

$$\rho\frac{dU_{i,t}}{d\tau} = \begin{array}{c} -\frac{1}{(1-\tau)} + \frac{\omega\psi}{\tau} + \frac{\psi}{\eta}\frac{\rho}{\left\{g+\psi\left[(1-\varphi)g+\rho\right]\right\}\left[(1-\varphi)g+\rho\right]}\frac{dg}{d\tau} \\ -\left[\frac{\varphi\rho}{\left[(1-\varphi)g+\rho\right]^2} + \frac{1}{\eta}\frac{\rho}{\left[(1-\varphi)g+\rho\right]^2}\right]\frac{dg}{d\tau} + \frac{1}{\rho}\frac{dg}{d\tau} \end{array}. \tag{15}$$

The first four terms on the right hand side of Eq. (15) capture the welfare effects of a change in abatement policy on consumption, leisure and environment. The last term captures the growth effect of a change in the policy. More precisely, starting with the first term on the right hand side of Eq. (15), this is unambiguously negative: it represents the consumption loss incurred when the abatement policy level is increased, as more resources are allocated to abatements. Next, the following term is positive. It represents the positive impact on welfare of a better environment. The remaining terms on the right hand side of Eq. (15) depend on the initial level of abatements. Specifically, the third and fourth term on the right hand side of Eq. (15) capture the effect of the change in the level of abatements on labour time and location choice. Its sign is ambiguous because the two effects discussed in the previous section are working in opposite directions. These effects carry on and apply to the last term on the right hand side which, keep in mind, describes the growth effect of a policy change.

Turning to the characterization of the welfare maximizing abatement policy level, we see directly from Eq. (15) that the welfare maximizing abatement policy level can theoretically fall above or below the growth-maximizing abatement policy level (i.e. $\tau^w > \tau^{\max}$ or $\tau^w < \tau^{\max}$). As shown in Appendix A, the outcome depends on whether $\varphi\psi > 1$ or $\varphi\psi < 1$. However, referring to economic intuitions, the case $\varphi\psi < 1$ appears the most plausible empirically. The reason is that $0 < \varphi < 1$ is the weight of material resources relative to existing human capital in the law of motion of human capital (Eq. (3)); and ψ is the weight on environmental quality in the preferences of individuals. For this latter, we can also expect to have $\psi < 1$ as the weight given to consumption is normalized to one (Eq. (4)) or even refer to the parameter calibration by Pautrel (2008) who sets $\psi = 0.01$. Therefore, for simplicity, in the remaining of the paper, we assume that $\varphi\psi < 1$ is verified so that we can state Proposition 3 itself illustrated by Fig. 3 in which we made use of the benchmark parameter values given in Table 1 assuming that the abatement policy verifies: $\tau \in (0;0.2)$.

Proposition 3: *Under the parameter restrictions* $\varphi + 1/\eta < 1$ *and* $\varphi\psi < 1$, *the welfare maximizing abatement policy level necessarily falls below the growth-maximizing abatement policy level. Formally, we have:* $0 < \tau^w < \tau^{\max} < 1$.

Proof: See Appendix A.

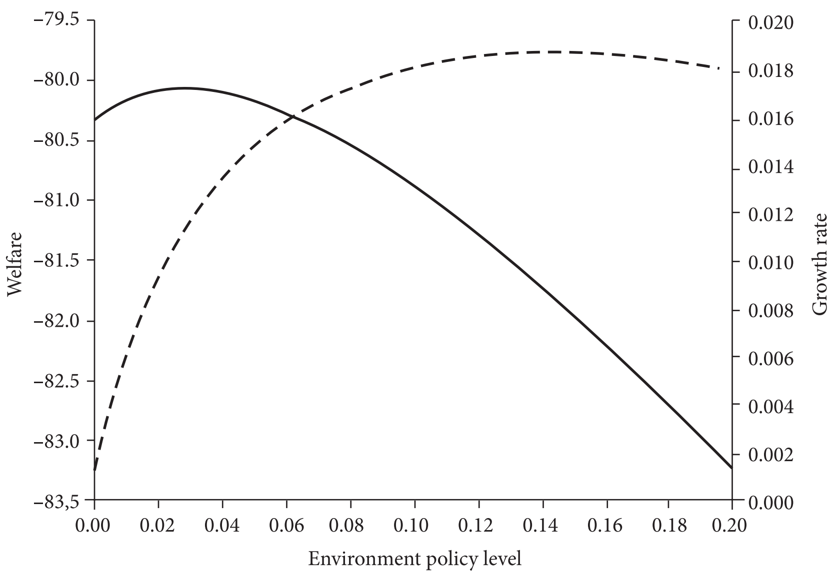

Fig. 3. Impacts of the environment policy on growth and welfare

Proposition 3 implies that if the government were to implement τ^w, and the level of abatements verifies $0 < \tau < \tau^w < \tau^{\max}$, as it is likely to be the case, the long-run welfare gains of a higher abatement policy are greater than the losses and total welfare increases. In other words, in the long-run there is no trade-off between welfare and growth if the abatement policy is not too stringent. That is, in line with the literature dealing with environmental issues and human capital accumulation, tighter abatement policies not only boost growth and environmental quality, but they are also welfare improving (van Ewijk, van Wijnbergen 1995; Oueslati 2002).

The interesting implication of Proposition 3, thereby the contribution here, is to show that, in choosing τ^w instead of $\tau^{\max}$, individuals are better off with greater pollution and lower growth than those which are possible to attain. The reason is that it allows them to benefit from more leisure given that their labour supply and distance of commuting to work are reduced. This result cannot be found in the standard literature in which agents are assumed identical and where housing location is not a choice variable. The underlying implication of Proposition 3, then, is to point out that the abatement policy can be an important factor affecting the shape of a city, region or country as it acts through the choice of location of individuals who are themselves differentiated with respect to their preferences for leisure, i.e. their degree of motivation. Moreover, if we consider that any government's objective is to implement, τ^w, it is important to point out that the value of the welfare maximizing abatement policy level depends, inter alia, on the degree of heterogeneity (or motivation) across individuals. In this context, turning to the analysis of the impact of inequality on τ^w, we obtain Proposition 4:

Proposition 4: Under the parameter restriction $\varphi + 1/\eta < 1$ and $\varphi\psi < 1$, a greater level of heterogeneity across individuals, σ_b^2, is likely to have a negative effect on the welfare maximizing abatement policy level, τ^w, if this latter is sufficiently low compared to $\tau^{\max}$. In the case, where τ^w and $\tau^{\max}$ are close enough, however, the reverse is more likely to apply.

Proof: See Appendix A.

What Proposition 4 implicitly suggests, is that, in affecting the value of the welfare maximizing abatement policy rate, the degree of inequality across individuals induces a change in individuals' welfare. More specifically, the intuition is the following. When the welfare maximizing abatement policy rate is initially low compared to the growth-maximizing one, as it is likely to be case in a situation where individuals put a relatively high weight on welfare gains coming from an increase in the consumption level, then a greater degree of heterogeneity induces a reduction of investments in abatements which deteriorates the environment. On the other hand, when the welfare maximizing abatement policy rate is close enough to the growth-maximizing one, as it is likely to be case in a situation where individuals put a relatively high weight on long-run welfare gains, then a greater degree of heterogeneity induces an increase of investments in abatements which improves environmental quality.

We can relate these results to empirical observations. Let us effectively mention that the analysis of the impact of inequality on environmental quality has been the subject of intensive research which goes back, at least, to the seminal work by Kuznets (1955). We recall that the main conclusion is that inequality has an inverse U-shaped relation against per capita income level, and so does pollution (at least for some polluants). This is known as the Environmental Kuznets Curve – EKC (Grossman, Krueger 1995) and implies that countries are likely to experience severe increases in pollution at the same time that they are growing. Recently some authors such as Torras and Boyce (1998) and Magnani (2000) have re-investigated the EKC hypothesis. Although a clear cut answer does not seem to have emerged yet, it seems that the literature suggests that income inequality is more likely to go along with a worse environmental quality[10]. From the results depicted in Proposition 4, thus, we can argue that such observation comes from the fact that individuals are more likely to favour welfare gains in the form of greater consumption and leisure levels, thereby they are more likely to be willing to face a worse environmental quality (reduction of the abatement policy level). We can also note from Proposition 1 that, in such case, the labour supply and distance of commuting to work are more likely to increase as individuals choose to live further away from the firm. Such pattern of behaviour also induces individuals to accept to face higher levels of pollution.

Conclusions

In this paper, we analyzed the impact of environmental quality on individuals' location choices, long-term growth and welfare in a unified model of endogenous growth. For simplicity, we assumed that environmental quality, which is unequally spread across the city, is set by the government via the level of abatement policy on output production. One innovation of the paper has been the formalization of a trade-off between the distance of commuting to work in the form of a welfare cost induced by the reduction of leisure time it requires and environmental quality which has a positive effect on individuals' human capital accumulation.

Using this framework, we showed that a tighter abatement policy leads individuals to choose housing locations further away from the source of pollution emissions. We also obtained an inverted-U shape relation between the long-run level of growth and environmental quality. As it is standard in such case, we argued that, whether the relationship is positive or negative, depends on the balance between the positive effects of a tighter abatement policy which boosts the pace of human capital accumulation and its negative impact which leads to a resource-withdrawal effect which has a negative effect on human capital accumulation. From a welfare point of view, the numerical analysis of the model showed that individuals are likely to favour a lower abatement policy level than the growth-maximizing one. Thereby, it implies that individuals prefer lower growth and are willing to face lower environmental quality than those which are possible to achieve. The reason is that such behaviour allows them to benefit from a greater amount of leisure and consumption. Another interesting finding was to point out that the degree of heterogeneity across individuals plays an important role in the determination of the welfare maximizing abatement policy level. On this matter, from the data, we argued that higher inequality is more likely to induce a reduction of environmental quality.

For analytical tractability, and for the purpose of establishing a first set of relevant results, we chose a simple endogenous growth framework. Some extensions are possible. For instance, it would be interesting to extend further our analysis to the case where there is a separate health sector. This is a relevant issue as health provision entails several trade-offs such as between consumption or health and education. That is, it raises the question of

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[10] More precisely, the main result of Torras and Boyce (1998) is that, in low-income countries, greater income equality leads to lower levels of pollution. However, they also find that greater income equality can be associated with worse environmental quality if heavy-particle air pollution and dissolved oxygen in water bodies are taken into account. Similarly, Magnani (2000) shows that if countries have average or above-average per capita incomes, greater income equality has a positive effect on environment; on the other hand, in countries with below-average levels of per capita income, the results are not clear cut.

how these trade-offs would affect the link between location choices, education, inequality, environment and growth.

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APPENDIX A. PROOF OF PROPOSITION 1

Taking expectation of Eqs. (7) and (9), we obtain Eqs. (11) and (12). To obtain Eq. (13), we manipulate Eq. (10) to obtain:

$$g\left[(1-\varphi)g+\rho\right]^{\frac{\varphi}{1-\varphi}}h_{i,t}=\left(\frac{1}{\beta_i}\right)^{\frac{1}{\eta(1-\varphi)}+\frac{\varphi}{(1-\varphi)}}\left[\frac{g}{(1-\varphi)g+\rho}+\psi\right]^{\frac{1}{(1-\varphi)\eta}}\times$$
$$\left[\phi(\varphi A)^{\varphi}(1-\tau)^{\varphi}(\tau)^{\omega}\right]^{\frac{1}{1-\varphi}}\overline{h}_t\ .$$

Taking expectation of this expression yields Eq. (13).

Proof of Proposition 2

By use of Eq. (11), (12) and (13), we obtain:

$$\frac{d\bar{l}}{d\tau}=\frac{\varphi\rho\bar{l}}{\left[(1-\varphi)g+\rho\right](g+\rho)}\frac{dg}{d\tau};$$
$$\frac{d\bar{\alpha}}{d\tau}=\frac{\bar{\alpha}\rho}{\eta\left[(1-\varphi)g+\rho\right]g}\frac{dg}{d\tau},$$

where:

$$\frac{dg}{d\tau}=\left[\frac{\omega-(\omega+\varphi)\tau}{\tau(1-\tau)}\right]\left[\frac{(1-\varphi)}{g}+\frac{\varphi(1-\varphi)}{(1-\varphi)g+\rho}-\frac{1}{\eta}\frac{1}{\frac{g}{(1-\varphi)g+\rho}+\psi}\frac{\rho}{\left[(1-\varphi)g+\rho\right]^2}\right]^{-1}. \quad \text{(A1)}$$

Thus, Proposition 2 follows.

Proof of Proposition 3

Manipulation of Eq. (15) yields

$$\rho\frac{dU_{i,t}}{d\tau}=-\frac{1}{(1-\tau)}+\frac{\omega\psi}{\tau}+\left[\begin{array}{c}\dfrac{-(\varphi\eta+1-\eta)\rho^2+(1-\varphi)^2g^2\eta+2(1-\varphi)g\rho\eta}{\rho\left[(1-\varphi)g+\rho\right]^2\eta}\\ +\dfrac{\psi\rho}{\left\{g+\psi\left[(1-\varphi)g+\rho\right]\right\}\eta\left[(1-\varphi)g+\rho\right]}\end{array}\right]\frac{dg}{d\tau}.$$

Examination of this condition shows that the term in square brackets on the right hand side is necessarily positive under the maintained assumption $\varphi+1/\eta<1$ and that $dU_{i,t}/d\tau=0$ can happen if $dg/d\tau>0$ or $dg/d\tau<0$. Therefore, $\tau^w<\tau^{\max}$ or $\tau^w>\tau^{\max}$ can occur. Assuming that the growth-maximizing abatement policy is implemented, we obtain:

$$\rho\frac{dU_{i,t}}{d\tau^{\max}}=-\frac{1}{\left(1-\tau^{\max}\right)}+\frac{\omega\psi}{\tau^{\max}}.$$

In other words, given that $\tau^{\max}=\omega/\left(\omega+\varphi\right)$ (Proposition 2), we have that $dU_{i,t}/d\tau^{\max}<0$ (>0) if $\psi\varphi<1$ $(\psi\varphi>1)$. Thus, we conclude that $\tau^{w}<\tau^{\max}$ $(\tau^{w}>\tau^{\max})$ if $\psi\varphi<1$ $(\psi\varphi>1)$. Based on these results and the discussion given in the main text, Proposition 3 follows.

Proof of Proposition 4

Using Eq. (A1), we note that the welfare-maximizing abatement policy rate is solution of:

$$\frac{\left(1+\omega\psi\right)\tau^{w}-\omega\psi}{\omega-\left(\omega+\varphi\right)\tau^{w}}=\frac{\left[\begin{array}{l}\dfrac{-\left(\varphi\eta+1-\eta\right)\rho^{2}+\left(1-\varphi\right)^{2}g^{2}\eta+2\left(1-\varphi\right)g\rho\eta}{\rho\left[\left(1-\varphi\right)g+\rho\right]^{2}\eta}\\+\dfrac{\psi\rho}{\left\{g+\psi\left[\left(1-\varphi\right)g+\rho\right]\right\}\eta\left[\left(1-\varphi\right)g+\rho\right]}\end{array}\right]}{\dfrac{\left(1-\varphi\right)}{g}+\dfrac{\varphi\left(1-\varphi\right)}{\left(1-\varphi\right)g+\rho}-\dfrac{\dfrac{\rho}{\eta}}{\left\{g+\psi\left[\left(1-\varphi\right)g+\rho\right]\right\}\left[\left(1-\varphi\right)g+\rho\right]}}.$$

where we know that $dg/d\tau^{w}>0$ (Proposition 3). We can use the previous expression to determine the effects of inequalities, σ_{b}^{2}, on the level of the welfare-maximizing abatement policy. Defining the function:

$$\Upsilon(g)=\frac{\left[\begin{array}{l}\dfrac{-\left(\varphi\eta+1-\eta\right)\rho^{2}+\left(1-\varphi\right)^{2}g^{2}\eta+2\left(1-\varphi\right)g\rho\eta}{\rho\left[\left(1-\varphi\right)g+\rho\right]^{2}\eta}\\+\dfrac{\psi\rho}{\left\{g+\psi\left[\left(1-\varphi\right)g+\rho\right]\right\}\eta\left[\left(1-\varphi\right)g+\rho\right]}\end{array}\right]}{\dfrac{\left(1-\varphi\right)}{g}+\dfrac{\varphi\left(1-\varphi\right)}{\left(1-\varphi\right)g+\rho}-\dfrac{\rho/\eta}{\left\{g+\psi\left[\left(1-\varphi\right)g+\rho\right]\right\}\left[\left(1-\varphi\right)g+\rho\right]}},$$

where we recall that the value of the growth rate, g, is implicitly given by Eq. (13) and depends on τ^{w} and σ_{b}^{2}, we have:

$$\frac{\left(1+\omega\psi\right)\tau^{w}-\omega\psi}{\omega-\left(\omega+\varphi\right)\tau^{w}}-\Upsilon\left[g\left(\tau^{w},\sigma_{b}^{2}\right)\right]=0.$$

Applying the implicit function theorem, we obtain:

$$\frac{d\tau^{w}}{d\sigma_{b}^{2}}=\frac{\dfrac{\partial\Upsilon\left[g\left(\tau^{w},\sigma_{b}^{2}\right)\right]}{\partial g}\dfrac{dg}{d\sigma_{b}^{2}}}{\dfrac{\left(1-\varphi\psi\right)\omega}{\left[\omega-\left(\omega+\varphi\right)\tau^{w}\right]^{2}}-\dfrac{\partial\Upsilon\left[g\left(\tau^{w},\sigma_{b}^{2}\right)\right]}{\partial g}\dfrac{dg}{d\tau^{w}}},$$

where simulations show that we are likely to have:

$$\frac{\partial\Upsilon\left[g\left(\tau^{w},\sigma_b^2\right)\right]}{\partial g}>0 ,$$

if ψ is not too high and given that $\varphi+1/\eta<1$ implies $-\left(\varphi+1/\eta-1\right)>0$ and $dg/d\sigma_b^2>0$. Thus, the sign of $d\tau^{w}/d\sigma_b^2$ depends on the sign of $\omega\left(1-\varphi\psi\right)/\left[\omega-\left(\omega+\varphi\right)\tau^{w}\right]^2-\left\{\partial\Upsilon\left[g\left(\tau^{w},\sigma_b^2\right)\right]/\partial g\right\}dg/d\tau^{w}$ which can be positive or negative. Using economic intuitions and some numerical calibrations, Proposition 4 follows.

APPENDIX B. TRANSITIONAL DYNAMICS

In this Appendix, we characterize the transitional dynamics. To simplify the analysis, we make two restrictive assumptions. First, the share of income allocated to education is fixed throughout life and given by its steady-state value (Eq. (8)): $\varepsilon_{i,t}=\varepsilon$ for all $i\in\left[0,N\right]$ and at each moment. Second, we assume that individuals have the same initial endowment of human capital: $h_{i,0}=h_0$ for any $i\in\left[0,N\right]$. These restrictions will allow us to study, in a straightforward manner, the transition of individual relative human capital, $\tilde{h}_{i,t}=h_{i,t}/\overline{h}_t$, which is constant in steady state.

Using Eq. (3) along with Eqs. (7)–(9), we obtain:

$$g_{i,t}=\phi\left(\varphi\right)^{\varphi}\left[\left(1-\tau\right)A\right]^{\varphi}\left(\tau\right)^{\omega}\left(\frac{1}{\beta_i}\right)^{1/\eta+\varphi}\left[\frac{g}{\left(1-\varphi\right)g+\rho}+\psi\right]^{\frac{1}{\eta}}\left(\frac{g}{\left(1-\varphi\right)g+\rho}\right)^{\varphi}\left(\tilde{h}_{i,t}\right)^{-1+\varphi},$$

where $g_{i,t}=\dot{h}_{i,t}/h_{i,t}$ and g is the steady-state value of growth given by Eq. (13).

Using this expression, the dynamic equation of relative human capital, $\tilde{h}_{i,t}$, is given by:

$$\frac{\dot{\tilde{h}}_{i,t}}{\tilde{h}_{i,t}}=g_{i,t}-g_t=$$

$$\phi\left(\varphi\right)^{\varphi}\left[\left(1-\tau\right)A\right]^{\varphi}\left(\tau\right)^{\omega}\left(\frac{1}{\beta_i}\right)^{1/\eta+\varphi}\left[\frac{g}{\left(1-\varphi\right)g+\rho}+\psi\right]^{\frac{1}{\eta}}\left(\frac{g}{\left(1-\varphi\right)g+\rho}\right)^{\varphi}\left(\tilde{h}_{i,t}\right)^{-1+\varphi}-g_t ,$$

where $g_t=\dot{\overline{h}}_t/\overline{h}_t$. Taking a first order Taylor approximation of this expression around the steady state, we obtain:[11]

$$\frac{\dot{\tilde{h}}_{i,t}}{\tilde{h}_{i,t}}=-\left(1-\varphi\right)\phi\left(\varphi\right)^{\varphi}\left[\left(1-\tau\right)A\right]^{\varphi}\left(\tau\right)^{\omega}\left(\frac{1}{\beta_i}\right)^{1/\eta+\varphi}\left[\frac{g}{\left(1-\varphi\right)g+\rho}+\psi\right]^{\frac{1}{\eta}}\left(\frac{g}{\left(1-\varphi\right)g+\rho}\right)^{\varphi}\times ,$$

$$\left(\tilde{h}_i^{*}\right)^{-2+\varphi}\left(\tilde{h}_{i,t}-\tilde{h}_i^{*}\right)$$

[11] The first order Taylor approximation for g_t is always zero because $\varepsilon_{i,t}=\varepsilon$ for all $i\in\left[0,N\right]$ and at each moment and the relative amount of human capital is equal to unity for the average individual: $\overline{h}_t/\overline{h}_t=1$ at each moment.

which describes a stable process. The steady-state relative amount of human capital, $\tilde{h}_i^*$, for each individual i, $i \in [0, N]$, is such that:

$$\tilde{h}_i^* = \left[\frac{\phi(\varphi)^{\varphi} \left[(1-\tau)A\right]^{\varphi} (\tau)^{\omega}}{g} \right]^{\frac{1}{1-\varphi}} \left(\frac{1}{\beta_i} \right)^{\frac{\frac{1}{\eta}+\varphi}{1-\varphi}} \left[\frac{g}{(1-\varphi)g+\rho} + \psi \right]^{\frac{1}{\eta(1-\varphi)}} \left(\frac{g}{(1-\varphi)g+\rho} \right)^{\frac{\varphi}{1-\varphi}} .$$

To illustrate the adjustment, suppose that individuals i, $i \in [0, N]$, start with a human capital ratio lower (greater) than its steady state value, i.e. $\tilde{h}_{i,0}^* < \tilde{h}_i^*$ ($\tilde{h}_{i,0}^* > \tilde{h}_i^*$). In this case, along the transition, $\tilde{h}_{i,t}$ approaches its steady state value from below (above) and $\dot{\tilde{h}}_{i,t} / \tilde{h}_{i,t} > 0$ ($\dot{\tilde{h}}_{i,t} / \tilde{h}_{i,t} < 0$).

APPENDIX C. CES TECHNOLOGY FOR OUTPUT

In this Appendix, we assume that the output technology is now given by a CES function of the form:

$$Y_t = A\left[\int_0^N \left(l_{i,t} h_{i,t}\right)^{\kappa} d_i \right]^{1/\kappa},$$

where $0 < \kappa < 1$. In this new set-up, the representative firm solves: $\text{Max}\pi_{Yt} = (1-\tau)A \left[\int_0^N \left(l_{i,t} h_{i,t}\right)^{\kappa} d_i \right]^{1/\kappa} - \int_0^N w_{i,t} l_{i,t} h_{i,t} d_i$. The solution to this program is standard. It leads to the demand functions for skilled labour. After computations, we obtain:

$$w_{i,t} = \left(l_{i,t} h_{i,t}\right)^{\kappa-1} \Gamma_t,$$

where

$$\Gamma_t = \left[\frac{(1-\tau)Y_t}{\int_0^N \left(w_{k,t}\right)^{\kappa/(\kappa-1)} dk} \right]^{(1-\kappa)},$$

is taken as given as agents are assumed to be atomistic.

On individuals' side, the Current-Value Hamiltonian of their problem, is given by:

$$CVH_{i,t} = \ln\left[\left(1-\varepsilon_{i,t}\right)\left(l_{i,t} h_{i,t}\right)^{\kappa} \Gamma_t \right] - \beta_i \left[l_{i,t} + \frac{\left(\alpha_{i,t}\right)^{\eta}}{\eta} \right] + \psi \ln\left(\alpha_{i,t}\right) +$$

$$\omega\psi \ln(\tau) + \mu_{i,t} \phi \alpha_{i,t} (\tau)^{\omega} \left[\varepsilon_{i,t} \left(l_{i,t} h_{i,t}\right)^{\kappa} \Gamma_t \right]^{\varphi} \left(\bar{h}_t\right)^{1-\varphi},$$

where $\mu_{i,t}$ is the co-state variable associated to the human capital accumulation process. The solution to this problem, along with the transversality condition given by: $\lim_{t\to\infty} \mu_{i,t} h_{i,t} e^{-\rho t} = 0$, is given by:

$$\beta_i = \kappa\left[\frac{1}{l_{i,t}} + \mu_{i,t}\varphi\frac{\dot{h}_{i,t}}{l_{i,t}}\right];$$

$$\frac{1}{\left(1-\varepsilon_{i,t}\right)} = \frac{\mu_{i,t}\varphi\dot{h}_{i,t}}{\varepsilon_{i,t}};$$

$$\beta_i\left(\alpha_{i,t}\right)^{\eta} = \psi + \mu_{i,t}\dot{h}_{i,t};$$

$$\kappa\left[\frac{1}{\mu_{i,t}h_{i,t}} + \frac{\varphi\dot{h}_{i,t}}{h_{i,t}}\right] + \frac{\dot{\mu}_{i,t}}{\mu_{i,t}} = \rho .$$

As in the main text, focusing on the steady-state solution, we finally obtain:

$$l_i = \frac{\kappa}{\beta_i}\left[\frac{\rho+g}{\rho+g\left(1-\kappa\varphi\right)}\right];$$

$$\varepsilon = \frac{\kappa\varphi g}{\rho+g};$$

$$\left[\frac{\psi}{\beta_i} + \frac{\kappa g}{\beta_i\left[\rho+\left(1-\kappa\varphi\right)g\right]}\right]^{\frac{1}{\eta}},$$

which are similar to Eqs. (7)–(9). Moreover, the growth rate is solution of the following expression:

$$g^{1-\varphi}\left(\rho+g\right)^{\varphi(1-\kappa)}\left[\rho+g\left(1-\kappa\varphi\right)\right]^{\varphi\kappa} \times\left[\psi+\frac{\kappa g}{\rho+\left(1-\kappa\varphi\right)g}\right]^{-\frac{1}{\eta}} = \phi\varphi^{\varphi}\left(\kappa\right)^{\varphi(\kappa+1)}\left(\frac{1}{\beta_i}\right)^{\frac{1}{\eta}+\varphi\kappa}\left(\tau\right)^{\omega}\left[\frac{\Gamma_t}{\left(\overline{h}_t\right)^{1-\kappa}}\right]^{\varphi}\left(\frac{\overline{h}_t}{h_{i,t}}\right)^{1-\kappa\varphi}.$$

Taking expectation of this expression yields:

$$g^{1-\varphi}\left(\rho+g\right)^{\varphi(1-\kappa)}\left[\rho+g\left(1-\kappa\varphi\right)\right]^{\varphi\kappa} \times\left[\psi+\frac{\kappa g}{\rho+\left(1-\kappa\varphi\right)g}\right]^{-\frac{1}{\eta}} = \phi\varphi^{\varphi}\left(\kappa\right)^{\varphi(\kappa+1)}\left(\tau\right)^{\omega}\left(\frac{\Gamma_t}{\left(\overline{h}_t\right)^{1-\kappa}}\right)^{\varphi} \times\exp\left[-\left(\kappa\varphi+\frac{1}{\eta}\right)\overline{b}+\frac{\left(\kappa\varphi+\frac{1}{\eta}\right)^2}{\left(1-\kappa\varphi\right)}\frac{\sigma_b^2}{2}\right].$$

which has similar properties as Eq. (13).

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