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REBALANCING STATIC BIKE-SHARING SYSTEMS: A TWO-PERIOD TWO-COMMODITY MULTI-DEPOT MATHEMATICAL MODEL

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Abstract. In this paper, an Integer Linear Programming (ILP) has been developed for rebalancing the stations of a Periodic Bike Relocation Problem (PBRP) in multiple periods. The objective function of the mathematical model is reducing costs of implementing trucks, transportation between stations and holding bikes on trucks during rebalancing. The variables we are following them in this model are conducting the optimal route in several periods, using the most appropriate trucks for these routes, and determining the best program for loading/unloading bikes for stations. The distinguishing features of the proposed model are considering several bike types, several exclusive trucks and several time periods. Finally, a numerical example confirms the applicability of the proposed model.

Keywords: bike-sharing systems (BSS), periodic bike relocation problem (PBRP), mathematical modelling.

Introduction

Bike-Sharing Systems (BSS) allow individuals to rent a bike at automatic rental stations scattered around a city, use them for a short journey, and return them to any other station in that city (Raviv, Kolka 2013). BSS has grown rapidly in the past decade. Although the concept has been around since the 1960s, the number of cities offering BSS has increased from just a handful in the late 1990s to over 800 at the time of publication (DeMaio 2016). Compared to private automobiles, BSS offers a number of environmental and social benefits. These include reduction in energy use, air and noise pollution, and congestion levels on specific corridors and access routes to public transport stops (Martens 2004).

The global growth of BSS has spurred an enthusiastic response from transport researchers, which has led to a burgeoning of papers examining bike-sharing. In general, research issues in this field of study are classified into four categories based on their diversity. The first category regards to *bike-sharing usage and user preferences*. For example, Pfrommer *et al.* (2014) examined that weekday usage of BSS peaks between 7:00–9:00 and 16:00–18:00, while weekend usage is strongest in the middle of the day. Ahmed *et al.* (2010) explored BSS are busier in the warmer months. Other researchers have found casual users typically take longer trips than annual members (Buck *et al.* 2013). In addition, convenience is the major perceived benefit identified by bike-sharing users (Fishman *et al.* 2013). Bachand-Marleau *et al.* (2012) found Montreal respondents living within 500 meters of a docking station were 3.2 times more likely to have used bike-sharing. LDA Consulting (2013) identified that trip purpose can vary by residential location, age, gender, ethnicity and whether the member has a car available for their use.

In the second category, researchers deal with *barriers to bike-sharing usage*. For example, finding from studies shows that bike-sharing members are more likely to live in close proximity to a docking station, in Montreal (Bachand-Marleau *et al.* 2012), London (Goodman *et al.* 2014), Melbourne and Brisbane (Fishman *et al.* 2014). In recently published research, Fishman *et al.* (2015) showed those who have not used bike-shared are considerably more sensitive to a lack of bike infrastructure than those who are bike share members. Subsequently, helmets have emerged as a contentious issue for BSS (Basch *et al.* 2014).

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In addition, users and would-be users have reported the lack of immediate sign-up is a barrier to usage (Fishman *et al.* 2012a).

Evaluating the impacts of BSS is the issue of some researches in the third category. For example, Shaheen *et al.* (2013) pointed that there are a number of purported benefits of BSS, including travel time saving, connection with public transport, health, air and noise pollution benefits. A study of a BSS in Shanghai showed that the majority of users are replacing walking and public transport (Zhu *et al.* 2013). Health impacts of BSS were dealt by Woodcock and Goodman (2014) on the London BSS. The researchers focused on three issues; physical activity, crashes and exposure to air pollution. Perceptions of safety (a lack of) have been established as a major issue for BSS generally, in Australia (Fishman *et al.* 2012b), the UK (Horton *et al.* 2012) and the USA (Starke 2002).

The last category of BSS study respects to *rebalancing problem*. Sayarshad *et al.* (2012) generated a multi-periodic mathematical model to optimize BSS design in small communities by determining minimum required bike fleet size with minimum unmet demands and unutilized bikes. Chemla *et al.* (2013) considered bike distribution between stations as a pick-up and delivery problem, and presented some algorithms for solving the rebalancing problem in BSS. Dikas and Minis (2014) formulated the static repositioning problem as a Mixed Integer Linear Programming (MILP) and presented two different models, one arc-indexed and the other time-indexed, whose objective functions include user satisfaction with the system and operating costs. Brake *et al.* (2007) presented four models for the bike-sharing rebalancing problem, considering a fleet of capacitated vehicles; they proposed customized branch-and-cut algorithms to solve the models. Schalekamp and Behrens (2013) developed mathematical programming models to determine the optimal daily allocation of bikes to stations in a BSS. Fu (2002) presented an inventory model suitable for the management of bike rental stations. Dell'Amico *et al.* (2014) presented four MILP models of BSS problem in which a fleet of capacitated vehicles is employed in order to re-balance the bikes with the objective of minimizing total cost.

This paper focuses on rebalancing Periodic Bike Relocation Problem (PBRP) where due to demand imbalance at some stations, pick-up or delivery bikes are impossible for some users. In some cases, the imbalance is persistent; e.g., relatively low return rates at stations located at the top of a hill. In other cases, the imbalance is temporary; e.g., suburban train stations are apt to face high return rates in the morning as commuters into the city drop off their bikes and high rental rates in the afternoon as commuters exit the train and begin to make their way home (Raviv, Kolka 2013). Satisfying user demand subject to such imbalances requires a dedicated fleet of trucks to regularly transfer bikes among stations. We refer to this activity as *rebalancing* bikes. Rebalancing of bikes in the PBRP involves routing decisions concerning the trucks, starting from and returning to the depots. The latter involves determining the number of bikes to be removed or placed in each station on each visit of the trucks. Ideally, the outcome of this operation would be to meet all demand for bikes and vacant lockers (Raviv *et al.* 2013).

In this paper, a mathematical model is developed for rebalancing PBRP based on recent published paper by Dell'Amico *et al.* (2014). In Dell'Amico *et al.* (2014), the model was able to determine the truck routes and the flow of load/unload bikes among stations based on demands and trucks' capacities. A maximum number of possible routes starting and ending depots is set and the models decides the optimal number of routes regarding Vehicle Routing Problem (VRP). The routes start and end at the same depot. In one formulation they have a second dummy depot that, however, is always the same one. In addition, m as the maximum number of possible routes was considered. The academic contribution of our paper is to further develop the model of Dell'Amico *et al.* (2014) by considering b bike types, e.g., VIP and normal bikes for various purposes, to enhance service level to users. In addition, the proposed model has developed in multi-depot situation. Rather than starting from one specific depot and ending to another depot, routes in the proposed model can be conducted from each depot and ends to the same depot or another one to rebalancing stations based on total costs. In continue, the mathematical model decides to choose the optimum number and type of trucks in each period. Rather than assuming one, k truck types were considered in their unique specifications of capacity, implementation cost and distance limitation. The capacities are defined for each bike type and this separated capacities are not be raped by another bike types. On the other hand, the capacity is divided per type and separated sections of the truck are dedicated to each bike-type. Finally, the proposed model was enabled to rebalance the stations' demands in multi periods.

The rest of the paper is organized as follows. We first provide problem description and the mathematical model in section 1. A numerical example is described and the results of solving it are explained in section 2. In section 3, a comprehensive discussion is provided to show the applicability of the proposed model. Finally, conclusion and findings are presented in the last section.

1. Mathematical formulation

1.1. Problem description

The problem is defined for rebalancing a finite number of bike stations in a PBRP due to occur unpredictable demands for bikes in each station. Because of variations in demand's patterns and uncertainty in bike delivery destination, the inventory of bikes in stations would be imbalanced. Based on demand predictions, a transportation network is designed to rebalance bike stations during specified hours of day while the network has stopped working. The process of rebalancing starts through announcing the demands by each stations for each bike type for the rebalancing times. The PBRP centre gathers the data for each time periods and decides to conduct the routes and assign appropriate trucks with proper amount of bikes to rebalance the whole stations. A day usually contains two periods t for rebalancing one at noon and another one at night. As the rebalancing times are so limited (almost 30 minutes) and the number of stations are remarkable, it is common to use multiple trucks k. So, various trucks in different specifications is needed in a fleet of transportation to rebalance the PBRP. Therefore, the arcs, the flows of loading/unloading and the variable defining the use of appropriate trucks are the variables, which are determined through solving the mathematical model regarding objective function, constraints and parameters.

We are given a complete graph $G=(V,A)$, where the set of vertices $V=\{0,1,\ldots,n+1\}$ are partitioned into the depots (vertices 0 and $n+1$), and the stations, vertices $\{1,\ldots,n\}$. Several bike types can be considered in PBRP for different purposes. In this research, two bike types (commodities) are defined comprised of VIP bikes and normal ones. Difference in commodities may be derived from quality of bikes, type of use or gender applicant. Each station j has a demand of commodity b in period $t\left(q_{jt}^{b}\right)$, which can be either positive or negative. If $q_{jt}^{b}>0$, then j is a pickup node in period t where q_{jt}^{b} bikes must be removed; if $q_{jt}^{b}<0$, then j is a delivery node where q_{jt}^{b} bikes must be supplied in period t. The bikes removed from pickup nodes can either go to a delivery node or back to the depots. Bikes supplied to delivery nodes can either come from the depots or pickup nodes. Whole demands must be meet and demand for each commodity cannot be fulfilled by another commodity. A fleet of k non-identical trucks of capacity s_{k}^{b} is available at depots to rebalance the stations. Each depot can be considered as starting or ending node in each route.

The PBRP problem involves determining how to drive the most suitable trucks through the graph, with the aim of minimizing total cost containing costs of implementing trucks, transportation between stations and holding bikes on trucks during rebalancing. In addition, the following constraints must hold:

- each truck performs a route that starts and ends at depots and trucks are able to journey a finite amount of distance;
- each truck starts from depot empty or with some initial load in period t (i.e., with a number of commodity b that vary from 0 to $Qtot_{t}^{b}$);
- each station is visited exactly once and its demand is completely fulfilled by the truck visiting it;
- the sum of demands of the visited stations plus the initial load is never negative or greater than $Qtot_{t}^{b}$ in the route performed by a truck.

In our study, each demand q_{jt}^{b} is computed as the difference between the number of commodity b present at station i in period t when performing the redistribution, and the number of commodity b in the station in the final required configuration. Note that, we impose a station even with no demand (demand $q_{jt}^{b}=(0,0)$ in each period) must be visited, even if this implies that no bike has to be dropped-off or picked-up there. This case arises, for example, when the driver of the truck is supposed to check that the station is correctly working. The case in which stations with null demands have to be skipped can be simply obtained by removing in a pre-processing phase those stations from the set of vertices.

The fact that each truck is allowed to start its route with some bikes enlarges the space of feasible PBRP problem solutions, and allows obtaining a more flexible redistribution plan. Note also that we do not impose the sum of redistributed bikes to be null, and hence, there can be a positive or a negative flow of bikes on the depot. This consideration is useful to model cases in which some bikes enter or leave the depot for maintenance.

The traveling cost c_{ij} is computed in our case as the shortest length of a path in the road network connecting i and j, for $(i,j)\in\tilde{V}$. It is important to work on a directed graph, because all BSS we are aware of are located in urban areas, and thus one-way streets typically have a strong impact on the choice of the routes performed by the trucks during the redistribution.

In this section, we present an Integer Linear Programming (ILP) model for PBRP problem based on the above assumptions. The model notations, parameters and variables are presented in the next part.

1.2. Mathematical modelling

The proposed mathematical model, which is described as follows has developed based on the Dell'Amico *et al.* (2014).

1.2.1. Notations and parameters

The parameter's symbols and their definitions are presented as Table 1.

Table 1. The symbols and definitions of parameters

Symbol	Definition
$\tilde{V}$	set of vertices
$\tilde{V}_0$	set of vertices except the depots (stations 0 and $n+1$ are depots)
A	set of arcs
K	set of trucks
B	set of commodities (VIP and normal)
T	set of time periods
n	number of stations
s_k^b	capacity of truck k of commodity b
q_{jt}^b	demand for commodity b at vertex j in period t
c_{ij}	transportation cost of the arc (i,j)
p_k	initial cost of implementing truck k
β_k	the total distance restriction for truck k

1.2.2. Variables

The variables of the mathematical model are provided as Table 2.

Table 2. The symbols and definitions of variables

Symbol	Definition
x_{ijkt}	taking value 1 if arc (*i,j*) is used by truck *k* in period *t*
f_{ijkt}^{b}	flow over arc (*i,j*) for commodity *b* by truck *k* in period *t*
w_{kt}	taking value 1 if truck *k* is used in period *t*
$\hat{h}_{kt}$	taking value 1 if truck *k* presence at depot 0 in the end of period *t*
h'_{kt}	taking value 1 if truck *k* presence at depot *n* + 1 in the end of period *t*

1.2.3. The proposed model

The proposed mathematical model has one objective function in three parts. The first part minimizes the cost of transporting from one station to another. Simply, it is assumed that this cost has a direct relationship with the distances. The second part points to the implementation cost of employing each truck for rebalancing. As several trucks were considered in this research, this part were added to the basic research to select the most appropriate trucks:

$$\min Z = \sum_{i\in\tilde{V}}\sum_{j\in\tilde{V}}\sum_{K}\sum_{T} c_{ij}\cdot x_{ijkt} + \sum_{K}\sum_{T} p_k \cdot w_{kt}. \quad (1)$$

Eqs (2) and (3) impose that every stations in each period must be visited exactly once. Since maybe several routes must be conducted to rebalance the network in each period, this rule does not include the depots. Because it is possible not transferring right data in right time and some failure may be occurred in statistical reports, all the stations must be visited:

$$\sum_{K}\sum_{i\in\tilde{V}} x_{ijkt} = 1,\ \forall j\in\tilde{V}_0, T; \quad (2)$$

$$\sum_{K}\sum_{i\in\tilde{V}} x_{jikt} = 1,\ \forall j\in\tilde{V}_0, T. \quad (3)$$

Eq. (4) ensures that the whole number of trucks leaving each depot in each period must return to the depots including the same depot or other depots after rebalancing. This constraint enables conducting routes in four master mode; starting and ending station 0, starting and ending station $n + 1$, starting from station 0 and ending to station $n + 1$ and starting from station $n + 1$ and ending to station 0:

$$\sum_{j\in V_0} x_{0jkt} + \sum_{j\in V_0} x_{n+1,jkt} = \sum_{j\in V_0} x_{j,n+1,kt} + \sum_{j\in\tilde{V}_0} x_{j0kt},\ \forall K, T. \quad (4)$$

Eq. (5) is the classical sub-tour elimination constraints, see, e.g., Gutin and Punnen (2007) that impose the connectivity of the solution. In this research, dimensions of *k* trucks and *t* period were attached to the basic constraint. Eq. (6) is replaced to the Eq. (5) to prevent containing sub-tours in fewer problem dimensions. u_{jt} is the sequence number of station *j* in its tour in period *t* (Bektas 2006):

$$\sum_{i\in S}\sum_{j\in S} x_{ijkt} \le |S| - 1,\ \forall |S| \subseteq \tilde{V}_0,\ S \ne \varnothing, K, T; \quad (5)$$

$$u_{it} - u_{jt} + n\cdot\sum_{K} x_{ijkt} \le n - 1,\ \forall i,j\in V_0, T. \quad (6)$$

Eq. (7) appoints the balance of the flows on each station entering and leaving bikes within rebalancing that must be exactly equal to the demands for each commodity in each time period:

$$\sum_{K}\sum_{i\in\tilde{V}} \left(f_{jikt}^{b} - f_{ijkt}^{b}\right) = q_{jt}^{b},\ \forall j\in\tilde{V}_0, T, B. \quad (7)$$

The total load leaving the initial depot should be in any case non-negative, and moreover, in case $Qtot_t^b$ (summation of all stations' demands for each commodity at period *t*) takes a negative value, it should be not lower than this value. This fact is imposed by Eq. (8). Similarly, Eq. (9) states that the total load entering the final depot is in any case non-negative, and not lower than the sum of all demands in case this is positive:

$$\sum_{K}\sum_{j\in\tilde{V}_0}\left(f_{0jkt}^{b} + f_{n+1,jkt}^{b}\right) \ge \max\left(0, -Qtot_t^b\right),\ \forall T, B; \quad (8)$$

$$\sum_{K}\sum_{j\in V_0}\left(f_{j,n+1,kt}^{b} + f_{j0kt}^{b}\right) \ge \max\left\{0, Qtot_t^b\right\},\ \forall T, B. \quad (9)$$

Eq. (10) imposes lower and upper bounds on the flows on each arc, and make these bounds as tight as possible by considering whether or not an arc is travelled by a truck. This values are limited depending on demands and truck capacities:

$$\max\left\{0, q_{it}^{b}, -q_{jt}^{b}\right\} x_{ijkt} \le f_{ijkt}^{b} \le \min\left\{s_k^b, s_k^b + q_{it}^{b}, s_k^b - q_{jt}^{b}\right\} x_{ijkt},\ \forall i,j\in A, K, T, B. \quad (10)$$

Eq. (11) ensures that each arc is traversed at most once by whole trucks in each period. Because *k* numbers of trucks are considered in this model, this constraint was added to the basic model:

$$\sum_{k} x_{ijkt} \le 1,\ \forall i,j\in A, T. \quad (11)$$

Eqs (12) and (13) guide bikes to be flown by containing a route of certain truck in each period. For example, if a route has covered with a special truck, it is not possible to be continued with another truck. Due to have several options for rebalancing by trucks and depots, this constraint is added to the model to reduce the space of feasible problem and remove the unrealistic cases in using trucks for routes:

$$x_{ijkt} + \sum_{K} x_{jikt} + \sum_{i'} \sum_{k'} x_{ji'k't} \le 1,$$
$$\forall j \in \tilde{V}_0, i, K, T,\ i' \ne i \text{ and } j' \ne j; \quad (12)$$

$$\sum_{j \in \tilde{V}} x_{ijkt} \le 1,\ \forall i \in \tilde{V}, K, T. \quad (13)$$

Eq. (14) considers a restriction of distance travelled by each truck. Because, some limitations such as petrol consummation or motor capacity compel trucks to travel moderately. The maximum distance for trucks on this kind of problems has already been considered, see e.g., Dell'Amico *et al.* (2016). This viewpoint can effect conducting routes for rebalancing stations:

$$\sum_{i \in \tilde{V}} \sum_{j \in \tilde{V}} c_{ij} \cdot x_{ijkt} \le \beta_k,\ \forall K, T. \quad (14)$$

Eq. (15) emphasizes that each truck must be employed at most once in each time period. This employing rule naturally causes at most two visits of depots comprised of the same or different depot for starting and ending:

$$\sum_{j \in \tilde{V}_0} x_{0jkt} + \sum_{j \in \tilde{V}_0} x_{j0kt} + \sum_{j \in \tilde{V}_0} x_{n+1,jkt} +$$
$$\sum_{j \in \tilde{V}_0} x_{j,n+1,kt} \le 2,\ \forall K, T. \quad (15)$$

Eq. (16) uses a binary variable to select the suitable trucks and consider the cost of implementing them:

$$\sum_{i \in \tilde{V}} \sum_{j \in \tilde{V}} x_{ijkt} \le (n-1) \cdot w_{kt},\ \forall K, T. \quad (16)$$

Eqs (17) and (18) use dependent variables of $\hat{h}_{kt}$ and h'_{kt} to communicate among trucks during time periods. These variables accept 0 or 1 values in which 1 indicates the presence of corresponding truck in the related depot at the end of the time periods:

$$\hat{h}_{kt} = \hat{h}_{k,t-1} + \sum_{j \in \tilde{V}_0} x_{j0kt} - \sum_{j \in \tilde{V}_0} x_{0jkt},\ \forall K, T; \quad (17)$$

$$h'_{kt} = h'_{k,t-1} + \sum_{j \in \tilde{V}_0} x_{j,n+1,kt} - \sum_{j \in \tilde{V}_0} x_{n+1,jkt},\ \forall K, T. \quad (18)$$

Eqs (19) and (20) ensure a situation in which only trucks that are in each depot at the end of the previous period can begin in the coming period:

$$\sum_{j \in \tilde{V}_0} x_{0jkt} \le \hat{h}_{k,t-1},\ \forall K, T; \quad (19)$$

$$\sum_{j \in \tilde{V}_0} x_{n+1,jkt} \le h'_{k,t-1},\ \forall K, T. \quad (20)$$

Eq. (21) introduces the boundary of the variables:

$$x_{ijkt}, w_{kt} \in \{0,1\},\ f^b_{ijkt} \ge 0. \quad (21)$$

2. Computational results

In this section, a numerical example is used based on a real-world instance provided by Dell'Amico *et al.* (2014) from Bari, Italy to show the applicability of the proposed model. There is a PBRP with 13 bike stations (contain 2 depots) and the goal is to rebalance the pre-determined demands of these stations for each commodity at two time periods (at 12:30 and 23:30). There are two commodities including VIP (type 1) and normal (type 2) bikes in each station and demands for each of them are independents. All demands must be meet and demands for each commodity cannot be fulfilled by another type. In addition, there are four trucks in different capacities, implementing costs and distance restriction to rebalance the stations. The trucks only start at node 1 or 13 (depots) in an appropriate initial bikes and end to the depots passing stations. At the beginning of the rebalancing, trucks 1 and 2 are at node 1 and trucks 3 and 4 are at node 13. Tables 3–5 show the parameters value for the numerical example.

The proposed mathematical model was solved by *Lingo 9.0* software in a dual-core system with CPU 3.0 GHz and 4 GB RAM Global optimal solution found with the objective value of 836300 after spending around 758 seconds CPU runtime. The best routes in each period were conducted and sequence of visits for each route was determined. The most suitable trucks were chosen regarding appropriate capacities and were assigned to the right routes.

Table 3. Costs of transportations between bike stations

Origin i	Destination j												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	–	2800	2100	1700	1100	3900	1300	3700	3300	3800	1700	2000	600
2	3000	–	1200	1200	2100	1400	3800	2000	1800	1400	1700	1600	2500
3	1800	1400	–	800	900	2400	2600	3300	2400	2400	1000	400	1300
4	1900	1200	600	–	1000	2200	2900	2900	2800	2200	600	1000	1400
5	1400	2100	1300	900	–	3100	2300	3800	2800	3100	1000	1300	1100
6	4600	2100	2800	2900	3700	–	5400	1600	2900	1000	3400	3200	4100
7	1600	3300	2500	2100	1200	4200	–	5000	3800	4200	2200	2400	1300
8	3600	1900	2900	2900	3800	1000	3800	–	1300	2000	3400	3100	3500
9	3100	1600	2600	2600	2800	1800	3400	1500	–	2700	3100	2000	3000
10	4200	1500	2700	2300	3200	600	4500	1500	2000	–	2800	3000	3700
11	1600	1700	1000	600	400	2700	2600	3400	2900	2700	–	900	1200
12	2500	1000	700	1100	1600	2400	3300	2900	2600	2400	1200	–	2000
13	600	2600	1800	1400	1200	3500	1600	4300	3000	3500	1500	1700	–

Table 4. The demand for each type of bikes in time periods

Origin i	Type 1		Type 2	
	t = 1	t = 2	t = 1	t = 2
1 (depot)	0	0	0	0
2	–1	5	–3	5
3	–3	1	–3	4
4	–1	3	2	1
5	–3	–4	–1	–5
6	1	1	–2	1
7	–4	3	4	–3
8	–2	–1	2	4
9	–1	0	–4	–4
10	–5	2	1	1
11	–1	3	–2	5
12	5	–1	–3	–3
13 (depot)	0	0	0	0

Table 5. First position, implementing costs, distance restrictions and capacities for trucks

Truck k	First position	Implementing cost p_k	Total distance restriction b_k	Capacity s_k^b	
				s_k^1	s_k^2
1	Depot 1	200000	11000	15	15
2	Depot 1	250000	13000	20	20
3	Depot 2	200000	11000	15	15
4	Depot 2	250000	13000	20	20

On the other hand, the most proper amount of initial commodities starting from depots and deliverable commodities ending to the depots were determined. Finally, the optimized program of pick-up and drop-off for each commodity in each station were determined. Figures 1 and 2 schematically show the optimized solution in time periods 1 and 2.

As it can be seen from Figures 1 and 2, results show that different routes and various trucks were determined in the optimized solution for rebalancing PBRP in two periods. In both periods, trucks 1 and 3 were employed to rebalance the stations. Truck 1 starts from the station 1 (depot 1) and rebalances the stations 7, 5, 12 and 3, respectively, and ends to the station 13 (depot 2). This truck is also used in period 2 starting from station 13 and ending to station 1. Truck 3 is the another truck employed in period 1, which starts from station 13 passing stations 9, 8, 6, 10, 2, 4 and 11, and ends to the station 13. This truck is also employed in period 2 starting from and ending to the station 13 passing stations 7, 3, 12, 2, 4, 11 and 15, respectively. Totally, the number of 26 and 8 bikes are picked up from the depots, also, 2 and 26 bikes are delivered to the depots in periods 1 and 2.

3. Discussion

Although a numerical example was considered in this paper to explain the efficiency of the proposed mathematical model, discussion on the optimized solution from different angles illustrates the features of the model and highlights its strengths and weaknesses. In this section,

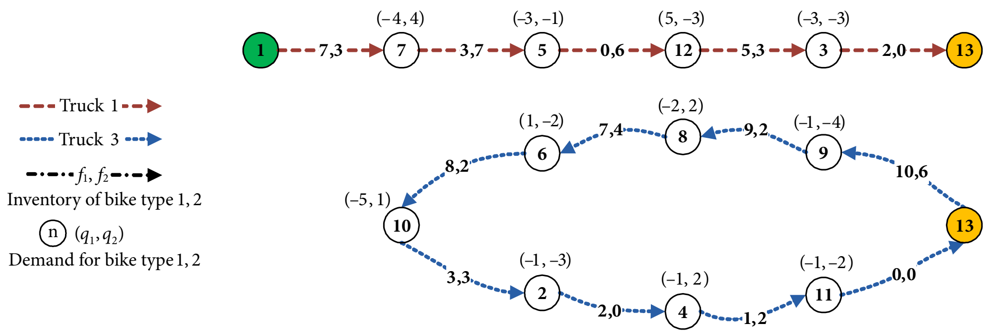

Figure 1. The final solution for time period 1

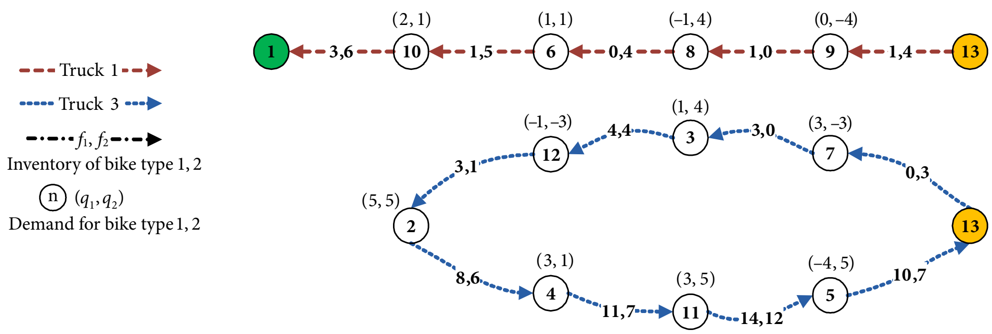

Figure 2. The final solution for time period 2

several analysis are done to discuss more about the results and underline how the final results obtained. In the first part, some important steps to reach the final solution are tracked. In continue some discussions are done through some targeted changes in the model and parameters.

As it is obvious, demand of each station and consequently visits program have a significant impact on decisions related to routing and select the suitable trucks for rebalancing. Therefore, the first analysis on the results is designed so that what would be the optimal solution if truck routing is only based on the transportation cost c_{ij} regardless of stations' demands. In this case, the objective is only finding the shortest path from a depot to another passing all stations. Figure 3 shows the final solution based on the shortest possible path.

Considering distance restriction β_k may impact on the final solution. If this limitation (Eq. (15)) is withdrawn from final model, the results show that the final solution is affected. The objective function in this case equals to the amount of 428410, which is better than the original problem because one truck is employed for each period. This means that the truck distance restriction play a decisive role in determining the optimal solution. Figure 4 shows the optimal solution in this case.

In this part of discussion, several numerical examples expanded from available datasets are provided. In our attempt to solve real-world instances, we used real data from Dell'Amico *et al.* (2014) and expanded the data based on the model. The cities included in our study are: Bari, Bergamo, Parma, Reggio Emilia, Treviso and La Spezia in Italy. We also used some randomly generated instances to show the quality of the proposed model. We generated 9 instances in total that are summarized in Table 6 and results are presented in Table 7. All instances are available in supplementary file.

Conclusions

The development of public urban transportation systems have a strong role in modern societies. Reduce noise and air pollution, reduce congestion level, reduce the cost of transportation of citizens and increase safety are just some of the notable points in the use of these systems. PBRP is one of these systems that although have designed since more than 50 years ago, its prosperity has recently started around the world. Meanwhile, this issue has attracted the attention of researchers more and more to help optimize the use of these systems.

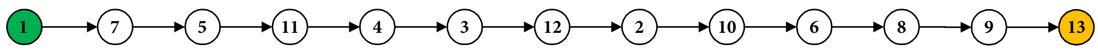

Figure 3. The simple routing based on the shortest possible path

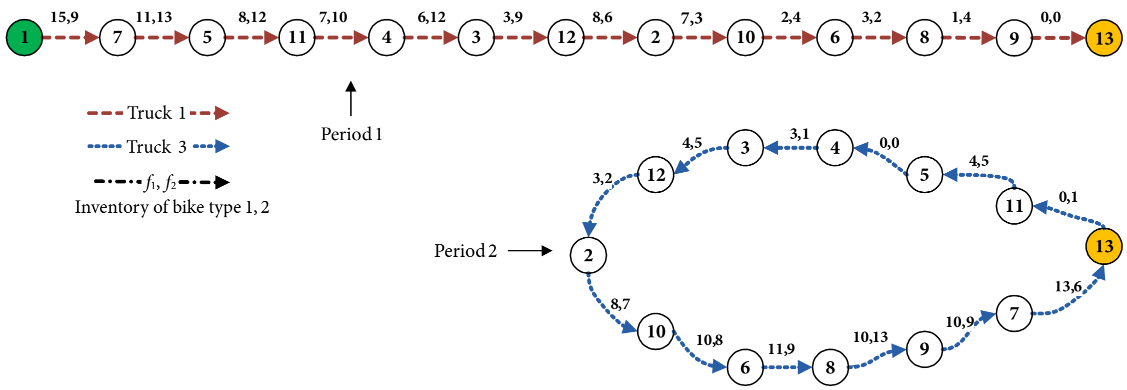

Figure 4. The solution not considering distance restrictions

Table 6. The instances of expanded datasets and numerical examples

City	$\lvert V\rvert$	min{q_i}	avg{q_i}	max{q_i}	dev{q_i}	min{c_{ij}}	avg{c_{ij}}	max{c_{ij}}	dev{c_{ij}}
NE.1	10	–10	0.06	12	5.74	139	392.24	874	157.06
NE.2	11	–7	–0.77	5	3.14	139	374.98	616	112.62
NE.3	12	–9	0.55	11	6.50	220	559.35	879	166.75
Bari	13	–5	–0.43	5	3.17	400	2283.97	5400	1067.62
Reggio Emilia	14	–10	–0.08	10	5.39	300	2095.05	5500	1110.44
Bergamo	15	–12	0.25	12	6.57	100	1532.86	3200	631.94
Parma	15	–10	–0.04	9	4.59	200	3121.48	8800	1857.86
Treviso	18	–8	0.18	8	4.27	340	3510.99	6462	2133.37
La Spezia	20	–7	0.23	9	4.24	193	2525.90	4987	1291.00

Table 7. The instances of expanded datasets and numerical examples

City	Time [sec]	Objective function	Truck: tours (period 1)	Truck: tours (period 2)
NE.1	412	404338	**3**: 10–7–5–8–9–4–6–2–3–1	**1**: 1–3–2–6–4–9–8–5–7–10
NE.2	450	404994	**3**: 11–10–2–6–4–3–9–7–5–8–11	**3**: 11–10–2–6–4–3–9–7–5–8–11
NE.3	484	408403	**3**: 12–11–7–10–3–8–2–6–9–4–5–12	**3**: 12–11–6–9–4–3–8–2–7–10–5–12
Bari	552	528100	**2**: 1–7–5–11–4–3–12–2–10–6–8–9–1	**2**: 1–7–5–11–4–3–12–2–10–6–8–9–13
Reggio Emilia	858	527200	**4**: 14–3–11–5–7–4–2–12–13–10–8–6–9–14	**4**: 14–6–9–8–3–2–12–13–10–5–7–11–4–14
Bergamo	1050	426500	**1**: 1–9–4–14–10–3–11–7–2–13–8–12–5–6–1	**3**: 15–10–3–7–2–13–8–12–9–14–4–6–5–11–15
Parma	1116	502418	**1**: 1–14–6–7–4–9–12–2–8–5–3–13–10–11–1	**2**: 1–6–7–4–14–11–9–12–2–8–5–3–10–13–1
Treviso	1387	458088	**3**: 18–5–14–10–12–13–16–17–8–11–6–9–7–15–2–4–3–18	**3**: 18–5–14–10–12–13–16–17–8–11–6–9–7–15–2–4–3–1
La Spezia	2119	440274	**1**: 1–9–7–12–8–17–10–18–15–4–3–6–5–13–11–2–16–14–19–1	**1**: 1–16–2–14–19–9–7–12–8–11–6–4–3–17–10–18–15–5–13–20

In this paper, we look at the PBRP from the field of rebalancing bike stations viewpoint. A mathematical model has developed for rebalancing the stations, which distribute various commodities in several time periods implementing special identical trucks. Some assumptions have considered in this research, which represent the framework of the proposed model as follows:

- the distance between stations was used as an estimate on the transportation cost;
- all the stations' demands must be covered in the relative period; lack or surplus for bikes has not defined.
- demands for each bike type cannot be fulfilled by other types;
- all stations must be visited exactly once in each period whether they had demands or not;
- all visits and loading/unloading program for rebalancing are done in the prescribed limited time;
- simply assumed that required trucks are available in each depot for whole time periods;
- each truck has unique space for each bike type and each bike must be put on its defined space;
- there are enough bikes to the required size in the depots and trucks have no problem to pick up them;
- there is enough space in depots for storing additional trucks and bikes.

In order to do future researches and develop the proposed mathematical model some suggestions are stated as follows:

- develop mathematical models through eliminating some of the mentioned assumptions;
- in this paper, a numerical example was used; it would be worthwhile if the model be implemented in a case study;
- the numerical example in this paper was solved by *Lingo 9.0* software and an exact solution was obtained; this problem is NP-hard and if this model is considered for real cases, meta-heuristic methods should be used to solve the problem;
- the present model is a single-objective to reduce the costs, while, multiple objectives including determining the level of service, sustainability in transportation, etc. can be considered.

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