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Original Article

DESIGNING A SUSTAINABLE CLOSED-LOOP SUPPLY CHAIN USING ROBUST POSSIBILISTIC-STOCHASTIC PROGRAMMING IN PENTAGONAL FUZZY NUMBERS

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Highlights:

- design of closed-loop supply chain under stochastic and cognitive uncertainty;
- offering a novel RSP approach using pentagonal numbers;
- presenting possibility necessity and credibility criteria for pentagonal numbers;
- evaluating the RSP approach in the stone paper industry;
- proposing trade-offs between objective functions considering the decision risk.

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Abstract. The lack of information and hybrid uncertainties in Supply Chain (SC) parameters affect managerial decisions. It is inevitable to consider random uncertainty based on fuzzy scenarios and cognitive uncertainty to model a Sustainable Closed-Loop SC (SCLSC) problem. Using Pentagonal Fuzzy Numbers (PFNs) has higher comprehensiveness and accuracy than triangular and trapezoidal fuzzy numbers due to taking into account higher uncertainty, less lack of information, and taking into account maximum subjectivity Decision-Makers (DMs). There is a gap in the literature regarding the use of PFNs in SCLSC problems. This research presents a new model using PFNs to solve deficiencies in stochastic-possibilistic programming. Developing a Robust Stochastic-Possibilistic (RSP) based on PFNs under fuzzy scenarios, presenting measures of necessity, possibility, and credibility for making decisions founded on different levels of DMs' risk, and proposing global solutions through providing linear programming models are the main innovations and contributions of the present research. An actual case study evaluates the presented approach to reduce the cost and carbon pollution in the stone paper SC. In the suggested method, trade-offs could be formed between the mean of objective functions and risk by modifying the robustness coefficients. According to the proposed approach, an optimal value of confidence is specified. Additionally, robustness deviations are controlled in the model, which results in more accurate and reliable results. Numerical simulations confirmed the efficacy of the robust approach proposed.

Keywords: closed-loop supply chain, possibilistic programming, fuzzy scenarios, robust approach, sustainable design, stochastic programming.

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Notations

Abbreviations:

CC – collection centre;
CLSCND – closed-loop SCND;
DC – distribution centre;
DM – decision-maker;
IFPS – interactive fuzzy programming solution;
MC – manufacturing centre;
NIS – negative ideal solution;
PFN – pentagonal fuzzy number;
PIS – positive ideal solution;
RC – recovery centre;
RS – recycling site;
RSP – robust stochastic-possibilistic;
SC – supply chain;

SCLSC – sustainable closed-loop SC;
SCND – SC network design;
SD – standard deviation;
SPP – stochastic possibilistic programming;
TH – Torabi–Hassini.

Variables and functions:

c – the related set to the customer centres c = 1, 2, ..., C;
$\widetilde{ch}_{qgdts}$ – the cost of carrying each unit of good q from the recovering location g to the DC d in time t and scenario s;
$\widetilde{co}_{qmdts}$ – the cost of carrying each unit of good q from the MC m to the DC d in the time t and scenario s;
$\widetilde{cp}_{qlgts}$ – the cost of carrying each unit of good q from the collecting location l to the recovering location g in time t and scenario s;
$\widetilde{cq}_{qclts}$ – the cost of carrying each unit of good q from customer place c to CC l in time t and scenario s;
$\widetilde{cs}_{qlnts}$ – the cost of carrying each unit of good q from the collecting place l to the RS n in time t and scenario s;
$\widetilde{cu}_{qdcts}$ – the cost of carrying each unit of good q from the DC d to the customer place c in time t and scenario s;
$\widetilde{cv}_{unvts}$ – the cost of carrying each item u from the RS n to the secondary market v in time t and scenario s;
d – the related set to the DCs d = 1, 2, ..., D;
$\tilde{d}_{qcts}$ – the amount of demand for good q by customers of place c in period t and scenario s;
$\tilde{d}_{uvt}$ – demand for recycled items u by the secondary market v in time t;
$\tilde{e}_{qclts}$ – the carbon emissions for transmitting each unit of good q from the customer location c to the CC l in time t and scenario s;
$\tilde{e}_{qdcts}$ – the carbon emissions for transmitting each unit of good q from the DC d to the customer location c in the time t and scenario s;
$\tilde{e}_{qdts}$ – the carbon emissions for processing each unit of good q in the DC d in time t and scenario s;
$\tilde{e}_{qgdts}$ – the carbon emissions for transmitting each unit of good q from the RC g to the DC d in time t and scenario s;
$\tilde{e}_{qgts}$ – the carbon emissions for recovering each unit of good q in the RC g in the time t and scenario s;
$\tilde{e}_{qlgts}$ – the carbon emissions for transmitting each unit of good q from the CC l to the RC g in time t and scenario s;
$\tilde{e}_{qlnts}$ – the carbon emissions for transmitting each unit of good q from the CC l to the RS n in time t and scenario s;
$\tilde{e}_{qlts}$ – the carbon emissions for processing each unit of good q in the CC l in time t and scenario s;
$\tilde{e}_{qmdts}$ – the carbon emissions for transmitting each unit of good q from the MC m to the DC d in time t and scenario s;
$\tilde{e}_{qmts}$ – the carbon emissions for the production of each unit of good q in the MC m in the time t and scenario s;
$\tilde{e}_{unts}$ – the carbon emissions for recycling item u in the RS n in time t and scenario s;
$\tilde{e}_{unvts}$ – the carbon emissions for transmitting each unit of item u from the RS n to the secondary market v in time t and scenario s;
$\widetilde{fc}_i$ – the cost of opening the ith centre ($i \in \{m, d, l, g, n\}$);
g – the related set to the RCs g = 1, 2, ..., G;
h_{qgdts} – the transmitted amount of good q from the recovering location g to the DC d in time t and scenario s;
h_{qmdts} – the transmitted amount of good q from the MC m to the DC d in time t and scenario s;
i – the related set to the considered centres ($i \in \{m, d, l, g, n\}$);
l – the related set to the CCs l = 1, 2, ..., L;
m – the related set to the MCs m = 1, 2, ..., M;
mc_i – if the ith centre is opened 1, otherwise 0 ($i \in \{m, d, l, g, n\}$);
n – the related set to the RSs n = 1, 2, ..., N;
o_{qmdts} – the transmitted amount of good q from the MC m to the DC d in time t and scenario s;
P_s – the possibility of the scenario s;
$\widetilde{pp}_i$ – the greatest capacity of the ith centre ($i \in \{m, d, l, g, n\}$);
p_{qlgts} – the transmitted amount of good q from the CC l to the RC g in time t and scenario s;
q – the related set to the type of product q = 1, 2, ..., Q;
q_{qclts} – the transmitted amount of good q from customer place c to CC l in time t and scenario s;
$\tilde{r}_{qcts}$ – the amount of returned goods q by customers of place c in time t and scenario s;
s – the related set of scenarios s = 1, 2, ..., S;
s_{qlnts} – the transmitted amount of good q from the CC l to the RS n in time t and scenario s;
t – the related set to the times t = 1, 2, ..., T;
u – the related set to the recycled items u = 1, 2, ..., U;
u_{qdcts} – the transmitted amount of good q from the DC d to the customer place c in time t and scenario s;
v – the related set to the secondary markets v = 1, 2, ..., V;
v_{unvts} – the transmitted amount of item u from the RS n to the secondary market v in time t and scenario s;
$\widetilde{w}_{qc}$ – the average percentage of returned goods q by customers of place c;
$\tilde{\gamma}_{unts}$ – the recycling cost of each item u in the RS n in time t and scenario s;
$\tilde{\partial}_{qlts}$ – the processing cost of each good q in the CC l in time t and scenario s;
$\tilde{\pi}_{qdts}$ – the processing cost of each good q in the DC d in time t and scenario s;
$\tilde{\rho}_{qmts}$ – the production cost of each unit of good q in MC m in time t and scenario s;
$\tilde{\tau}_{qgts}$ – the cost of recovering each unit of good q in RC g in time t and scenario s;
$\widetilde{\omega}_{qt}$ – the average failure rate of good q in time t.

1. Introduction

Recently, designing SCs by considering different real-world applications has been emphasized in the literature (Chen *et al.* 2023). It has also become essential and inevitable due to the issues of competitive pressure in international markets, customer expectations, continuous improvement of manufactured products, dynamic customer behaviour, and efficient supply and transportation processes (Hosseini Dehshiri, Amiri 2024). For practitioners of SC in the current era, SCND has become crucial due to its benefits and impact on enhancing the competitive advantage that it delivers (Seydanlou *et al.* 2023).

As a result of growing environmental concerns and competitive pressures to increase the life cycle of products and reduce their costs, researchers and industrial activists have begun to focus more on the SCLSC (Ghosh, Roy 2023). Due to the financial advantages of recovery and recycling and the concern for environmental sustainability and waste reduction, CLSCND has become increasingly significant for corporations (Zhou *et al.* 2023a). Sustainable CLSCND oversees an expansion in the profitability of the SC through the reduction of waste and prevents the withdrawal of products and the short life cycle of the product, and is emphasized due to the simultaneous attention to economic and environmental aspects (Seydanlou *et al.* 2023).

The complexity, dynamics, and extent of the SC network have led to uncertainty, which affects the performance of CLSCND (Izadikhah *et al.* 2021). Controlling uncertain parameters is one of the responsibilities of SCLSC management; challenges in sustainable CLSCND include uncertainties in supplying products or raw materials, distribution and production processes, demand estimates, and the number of product returns (Garai *et al.* 2021). The impact of uncertainty is very significant in sustainable CLSCND decisions with a strategic horizon, and it should be considered in network design. Various types of uncertainty have been investigated by researchers in sustainable CLSCND problems (Izadikhah *et al.* 2021).

Epistemic uncertainty is one type of uncertainty in sustainable CLSCND problems. In this state, the parameters are inadequate and ambiguous, the SCLSC problem has cognitive uncertainty, and possibilistic programming is used to face DMs without knowledge of cognitive uncertainty (Serrano-Guerrero *et al.* 2021). Possibilistic programming is flawed because it relies on taking into account expected values of vague parameters and the objective function (Zhou *et al.* 2023a). Some parameters have stochastic uncertainty in SCLSC modelling, so a stochastic fuzzy scenarios approach should be considered (Qiao, Chen 2023). One weakness of the possibilistic-stochastic approach is the lack of reduction possibilistic and scenario deviations (Hosseini Dehshiri, Amiri 2024). Robust programming was expanded to address the drawbacks of the stochastic-possibilistic approach (Zhang *et al.* 2023). In the robust approach, the solution's robustness should be considered from the optimality and feasibility robustness perspective, and the solution should not be sensitive to all values of non-deterministic parameters (Pishvaee *et al.* 2012; Zhou *et al.* 2023b). Robust optimization can reduce uncertainty even when there is little distributional data (Hosseini Dehshiri, Amiri 2024). The possibilistic-scenario deviations and the constraints un-fulfilment are controlled in a RSP approach (Hosseini Dehshiri *et al.* 2023). Thus, using a RSP approach allows SCLSC problems to take uncertainty into account.

Extensive, complex, sustainable CLSCND problems require simultaneous attention to fuzzy and robust aspects. Fuzzy numbers are a helpful factor in analysing hybrid uncertainty because they consider ambiguous terms and concepts in computations, and the outcomes heavily depend on the form of these numbers' membership functions (Veryard *et al.* 2023). Numerous domains, such as fuzzy process modelling, control theory, decision-making, and expert systems, use fuzzy numbers (Qahtan *et al.* 2023). Triangular, trapezoidal, and PFNs are frequently preferred because they have simple membership functions (Mondal, Mandal 2017). According to a review of studies, PFNs are preferable for considering hybrid uncertainty since they are efficient and convenient for high-performance computing and have piecewise linear membership functions that can only be fully represented by a small number of actual values (Báez-Sánchez *et al.* 2022). The PFNs are beneficial for DMs to analyse the result more accurately due to their ability to estimate imprecise parameters and consider uncertainty (Mondal, Mandal 2017).

The completed sustainable CLSCND research reveals that triangular and trapezoidal fuzzy numbers evaluate cognitive and random uncertainties in modelling and DMs' opinions. This is even though PFNs have not been used in modelling the SCLSC problem. Also, until now, PFNs have not been used in stochastic-possibilistic programming and robust approaches for hybrid uncertainty modelling in SCLSC. There is a shortage of research in this field. Thus, to address the current shortages, in this research, for the 1st time, the novel approaches of possibilistic and RSP are developed using PFNs for stone paper SCLSC in Iran. PFNs are used because they can achieve the most significant subjectivity of DMs compared to other fuzzy sets, such as trapezoidal or triangular fuzzy numbers. Also, in different conditions, DMs can consider different types of PFNs according to the problem definition and have higher accuracy than trapezoidal and triangular numbers, considering the uncertainty of the real world (Mondal, Mandal 2017). If DMs want to consider more uncertainty or lose less information, using PFNs is more appropriate than triangular or trapezoidal fuzzy sets. Therefore, this study uses PFNs to reduce ambiguity and lack of knowledge and to improve accuracy and comfort in considering uncertainty. The principal contributions and objectives of this research are as follows:

- considering the maximum subjectivity of DMs compared to triangular and trapezoidal fuzzy numbers by using PFNs for modelling SCLSC under hybrid uncertainty;

- introducing the novel possibilistic programming approach based on PFNs to solve the sustainable CLSCND problem under uncertainty and improve in reducing the degree of ambiguity and lack of information;
- developing a novel RSP approach for simultaneously considering cognitive and random uncertainties and paying attention to robustness in SCLSC modelling;
- presenting the measures of necessity, possibility, and credibility based on PFNs to consider uncertainty according to the different levels of DMs risk-taking and presenting linear models of RSP programming to obtain the optimal global solutions;
- evaluation of the presented RSP approach in the actual case for reducing the cost and environmental effects in SCLSC and robustness-sensitivity analysis, and comparison of the efficiency of the presented approaches;
- offering a realistic and flexible approach to make trade-offs among the sustainable objectives, including cost and environmental effects in the presented case, considering the risk level based on the range of optimistic-pessimistic preferences of DMs' opinions and values of robustness coefficients.

The construction of this research is as follows:

- the current Section 1 – introduction;
- the literature examines the usage of PFNs and SCLSC problems under uncertainty in the Section 2;
- the offered SCLSC problem is examined in the Section 3;
- the Section 4 introduces a novel possibilistic programming approach based on PFNs;
- the Section 5 proposes a novel RSP programming using PFNs;
- in the Section 6, applying the suggested procedure is evaluated in the actual study, and robustness analysis, sensitivity analysis, performance evaluation of the offered procedure, and insights are presented;
- in the Section 7, conclusions and scientific and practical suggestions are presented.

2. Literature review

Due to the advantages of PFNs compared to trapezoidal and triangular fuzzy numbers, such as providing more accuracy, reducing the lack of information, and the need to consider uncertainties in the SCLSC problem, in this section, at 1st, the conducted studies using PFNs are discussed. Then, the performed studies using possibilistic programming and RSP programming are classified and compared in the field of sustainable CLSCND, and the innovations of the current study are highlighted in comparison with the previous studies.

2.1. Applications of PFNs

Zadeh (1965) introduced the concept of fuzzy numbers. In the crisp set theory, each element is specified exactly and each element is placed inside or outside the set, while in the logic of the fuzzy theory, a set is specified with uncertain boundaries (Kazda *et al.* 2023). Due to the advantages and wide range of fuzzy numbers, various applications for these numbers have been presented in different fields (Amoozad Mahdiraji *et al.* 2018; Hosseini Dehshiri *et al.* 2024). Trapezoidal and triangular numbers are widely utilized in the literature; however, much information may be lost by DMs. Using PFNs, DMs can consider more uncertainty and maximum subjectivity in evaluation, and PFNs are more suitable than triangular and trapezoidal fuzzy numbers (Mary, Sangeetha 2016). Researchers looked into the concept of PFNs and pentagonal fuzzy matrices in this regard because there is uncertainty in many mathematical models in various sectors of science and industry (Panda, Pal 2015).

A category of related literature developed the theoretical framework and mathematical approaches for PFNs: Panda & Pal (2015) discussed PFNs 1st. Based on PFNs, the structure and fundamental characteristics of pentagonal fuzzy matrices were examined. This paper discussed some unique kinds of PFN matrices and their algebraic properties. Mondal & Mandal (2017) investigated PFNs and developed a type of PFN. Using linear and non-linear membership functions, they defined symmetric and asymmetric PFNs in fuzzy equations. They provided a numerical example to evaluate the proposed concept. Considering membership functions, Visalakshi & Suvitha (2018) developed a performance index for fuzzy queues utilizing the α-cut approach. The α-cut method was applied to linear PFNs, and a numerical example was investigated in this research. Srinivasan *et al.* (2021) investigated a finite source queue model based on PFNs. The arrival and service times were converted to definite values for PFNs using Pascal's triangular graded mean. The suggested method converts PFNs into deterministic values that can be applied to the limited resource queuing model. This procedure was evaluated through numerical examination. In this class of studies, only mathematical model development has been considered theoretically. Numerical examples have been used to assess the approach, and these studies have a shortage of introducing new applications in actual cases.

The new applications for using PFNs were investigated in some studies: by applying a linear membership function, Dhanamandand & Parimaldevi (2016) ranked PFNs. This research was applied to the multi-item, multi-objective inventory mode. In the research in a fuzzy condition, Anitha & Parvathi (2017) developed an EPQ model for products with stock-dependent demand rates. Using the linear membership function, they provided expected crisp values of PFNs in inventory control problems. Hemalatha & Annadurai (2023), using 4 types of triangular, trapezoidal, pentagonal, and hexagonal fuzzy numbers, developed an optimization inventory system with advanced payment in a fuzzy situation. This study reduced the overall cost, and numerical studies were provided to reveal the applications of the suggested procedure. The optimal fish manufacturing amount for perishable fish goods was examined by Kuppulakshmi *et al.* (2021) utilizing PFNs. An efficient method was presented to discover the amount of annual fish production and production to manage demand and supply to retailers based on PFNs. In a paper, Alharbi & El-Wahed Khalifa (2021) provided a straightforward method using PFNs for the flow-shop scheduling problem. The PFN ranking technique was utilized to specify the fuzzy flow scheduling problem. A numerical example was given. Chakraborty *et al.* (2022) investigated nonlinear pentagonal intuitionistic fuzzy numbers and classified them in different scenarios. A new technique was investigated to deal with nonlinear pentagonal intuitionistic fuzzy numbers. The implementation of the procedure was investigated in the EPQ problem, and numerical examples were used to evaluate the approach. In this class of studies, new applications using PFNs were introduced, but in most previous studies, numerical investigations were applied to evaluate the procedure, and the execution of the suggested procedure in real studies was less considered. On the other hand, the modelling of SCLSC issues was not investigated in this group of studies, and there is a deficiency of investigation in this field.

Therefore, according to the literature review, a group of studies focused on developing theoretical frameworks and mathematical models. In this group of studies, numerical examples were used to evaluate the proposed approach. There is a lack of implementing and using PFNs in actual cases to consider uncertainty. On the other hand, in a group of literature on the field of using PFNs, new applications using PFNs were investigated, and PFNs have not been used to model uncertainty in SCLSC problems. PFNs have higher accuracy than triangular and trapezoidal fuzzy numbers for considering uncertainty due to DMs' maximum subjectivity and ability to satisfy imprecise parameters. Therefore, there is a lack of research on using PFNs in sustainable CLSCND.

2.2. Robust optimization and stochastic-possibilistic programming in CLSCND

Serious environmental problems and economic benefits from reuse have made SCLSC the central issue of interest to researchers (Zhang *et al.* 2022). Hybrid uncertainty is a severe barrier in designing SCLSC. Researchers have recommended RSP programming to investigate hybrid uncertainty in CLSCND problems (Yousefi Nejad Attari *et al.* 2021). In this section, the performed studies utilizing robust optimization and stochastic-possibilistic programming are reviewed for the designing of SCLSC:

A category of research used robust optimization for CLSCND: Pishvaee *et al.* (2011) studied SC design under uncertainty. A robust optimization model under uncertainty was proposed. Deterministic number generation was used based on the uniform distribution for calculations. Numerical examinations were utilized in this paper. Finally, the robustness of the obtained solutions was compared and analysed. Ma *et al.* (2016) investigated CLSCND under uncertainty. This study used robust mixed integer nonlinear programming to design a closed-loop environmental SC network. Numerical examples were applied based on crisp numbers under different scenarios. Avakh Darestani & Hemmati (2019) conducted the design of SC of perishable goods under uncertainty. The robust optimization approach was used using deterministic numbers in this research. Hassanpour *et al.* (2019) focused on evaluating the impact of government policies on CLSCND to achieve optimal collection decisions. The effect of uncertainty was investigated in government regulations and SCLSC configuration using robust optimization. Numerical examples were used based on the generation of deterministic numbers in the uniform distribution range.

Nayeri *et al.* (2020) presented a mathematical procedure for CLSCND of water tanks considering sustainable factors. This study used robust fuzzy optimization based on trapezoidal fuzzy numbers to deal with uncertainty. Then, the problem was solved using goal programming, and the outcomes confirmed the suitability of the approach. Abdolazimi *et al.* (2020) investigated the CLSCND problem using the robust optimization approach. Deterministic data was used in the uniform distribution range to account for uncertainty. An examination was conducted in tire factories. Pei *et al.* (2022) studied the design of multi-period dual-channel SC. In this study, robust optimization was used based on trapezoidal numbers. Numerical examples were used to evaluate the proposed approach. Rouhani & Amin (2022) dealt with the design of organ transplant SC. This study used the robust optimization approach to reduce time and costs. The optimal solutions of the models were calculated and analysed in a case study. Gao *et al.* (2024) investigated dual-channel CLSCND in uncertainty conditions. This study used robust optimization to address the challenges of the appliance industry. The findings revealed that the increase in online consumers led to a decrease in warehouses and an increase in uncertain demand led to an increase in total costs. In this group of studies, only the robustness of the results was considered, and the cognitive and random uncertainties of the parameters were not investigated.

Furthermore, in this field, a shortage of studies is seen. On the other hand, PFNs were not used to consider uncertainty in this class of studies. Due to the consideration of the maximum subjectivity of DMs to consider uncertainty and increase accuracy in modelling SCLSC problems by PFNs, studies are deficient in this field.

Possibilistic programming and robust optimization were investigated in many studies: Pishvaee *et al.* (2012) discussed the design of SC social responsibility. The robust possibilistic approach was introduced based on the necessity criterion for trapezoidal fuzzy numbers. The industrial case was presented to show the application of the procedures. To procure raw materials for CLSCND, Ghahremani-Nahr *et al.* (2019) presented a facility location procedure. The robust possibilistic approach was used using necessity criteria and trapezoidal fuzzy numbers, and the results confirmed the procedure's performance. The economic and environmental performances of green SCLSC were modelled by Liu *et al.* (2021). Using credibility criteria for trapezoidal fuzzy numbers, they offered a robust possibilistic procedure. The results were analysed in the *Coca-Cola* factory in China. Habib *et al.* (2021) developed the SC of biodiesel based on animal fat. A possibilistic approach using robust optimization was presented with a credibility standard based on trapezoidal fuzzy numbers. The results showed the appropriateness of the procedure. Lahri *et al.* (2021) designed a sustainable SC network and used a possibilistic programming approach based on triangular fuzzy numbers to consider uncertainty, and numerical examples were utilized in this research. Gilani & Sahebi (2021) worked on the design of pistachio SC and its by-products. Based on the necessity and trapezoidal numbers, the robust possibilistic procedure was utilized to meet uncertainty. The efficient results of the suggested procedure were satisfactory. Foroozesh *et al.* (2022) addressed the SC design of perishable products. Based on credibility criteria for trapezoidal fuzzy, robust possibilistic programming was applied in this study. A case study in a food company verified the model's performance. Baghizadeh *et al.* (2022) studied the CLSCND under uncertainty. The possibilistic approach using robust optimization focused on the *Me*-standard in trapezoidal fuzzy numbers. The proposed approach was implemented in the case of greenhouse production in a fruit production company. Ghahremani-Nahr *et al.* (2022) addressed the integrated design of blood SC, considering economic and environmental aspects. Robust possibilistic programming focused on the necessary criterion in this study with trapezoidal fuzzy numbers. The outcomes confirmed the effectiveness of the procedure. Ghasemi *et al.* (2022) designed blood SC, and the robust-possibilistic procedure was utilized for trapezoidal fuzzy numbers. A case study was used in this research. Habib *et al.* (2022) studied the design of waste management in the SC, focusing on sustainability goals. The robust-possibilistic procedure was utilized on the *Me* in trapezoidal fuzzy numbers. This study helped policymakers in developing tactical and strategic plans for waste management. Babaee Tirkolaee *et al.* (2023) studied the blood SC network. The possibilistic procedure was used to model the problem. Then, an examination of the blood SC was used to demonstrate the effectiveness of the suggested procedure. The outcomes showed that the method used was appropriate. Only cognitive uncertainty was investigated in this group of studies, and random uncertainty was not considered in the modelling. On the other hand, triangular and trapezoidal fuzzy numbers were used for modelling in these studies, which have shortcomings in using PFNs in SCLSC problems.

Scenario-based RSP approach was considered in a group of studies by researchers for SCLSC modelling: Torabi *et al.* (2016) used the possibilistic-stochastic approach on the *Me* for CLSCND. In this study, triangular numbers were used for modelling. In a study, Dehghan *et al.* (2018) investigated CLSCND considering the RSP programming. This study used trapezoidal numbers to model the multi-product and multi-period problems. Farrokh *et al.* (2018) investigated the CLSCND problem under cognitive and stochastic uncertainties. This study used trapezoidal fuzzy numbers and credibility criteria for modelling. Atabaki *et al.* (2020) designed SCLSC using robust optimization and possibilistic-stochastic programming methods. This research uses triangular numbers to consider ambiguous parameters in numerical examples. Yu & Solvang (2020) offered a stochastic-possibilistic mathematical procedure based on flexible constraints for SCLSC design. Ala *et al.* (2024) investigated blood CLSCND to reduce blood products' total cost and delivery time. In this study, a robust possibilistic approach was considered for network design. The findings indicated improvements in overall delivery time and total cost. Guo *et al.* (2024) investigated the responsive CLSCND problem based on scenario-based robust possibilistic model approaches. Economic aspects, delivery time, and reliability were examined as indicators of responsiveness. Numerical investigation confirmed the applicability and appropriateness of the proposed approach. This research utilized triangular numbers and numerical investigations to evaluate the proposed approach. In this category of studies, triangular and trapezoidal fuzzy numbers were used for CLSCND problems. Research using PFNs to model SCLSC with uncertainty is lacking. In the following, the studies in applying robust and stochastic-possibilistic programming approaches are compared in Table 1.

2.3. Research gaps

Current literature on sustainable CLSCND indicates that PFNs have not been used for hybrid uncertainty. Therefore, there are research gaps in this area. Triangular and trapezoidal fuzzy numbers have been widely used in SCLSC literature, while DMs are losing more information considering triangular and trapezoidal fuzzy numbers than PFNs. If DMs want to consider more uncertainty or lose less information, PFNs are suitable. On the other hand, in previous studies, one criterion has been used to consider DMs' risk-taking level. The different perspectives of DMs, such as necessity, possibility, and credibility, have not been reviewed simultaneously. Also, the necessity, possibility, and credibility standards have not yet been developed in the robust possibilistic programming approach for PFNs. The literature showed that several studies utilized numerical examples to assess the effectiveness of the presented approach. This study proposes a new approach called RSP programming based on PFNs to address current research gaps. This approach for PFNs includes criteria for possibility, necessity, and credibility. A linear programming model based on PFNs solves the sustainable CLSCND problem and achieves global optimal solutions. A case study on stone paper SC is reviewed to assess the presented approach's effectiveness and analyse the outcomes. The outcomes are analysed based on DMs' risk-taking by the possibility, necessity, and credibility criteria.

Table 1. Comparison of the studies in CLSCND

Author(s)	Programming method				Measurement approach	Information form				Multi-product	Multi-period	Experiment	
	fuzzy	robust	possibilistic	stochastic		crisp	triangular fuzzy	trapezoidal fuzzy	pentagonal fuzzy			numerical	real case
Pishvaee *et al.* (2011)		✓			–	✓						✓	
Pishvaee *et al.* (2012)	✓	✓	✓		necessity			✓		✓			✓
Ma *et al.* (2016)		✓			–	✓				✓		✓	
Torabi *et al.* (2016)	✓	✓	✓	✓	*Me*		✓					✓	
Dehghan *et al.* (2018)	✓	✓	✓	✓	*Me*			✓		✓	✓		✓
Farrokh *et al.* (2018)	✓	✓	✓	✓	credibility			✓			✓	✓	
Ghahremani-Nahr *et al.* (2019)	✓	✓	✓		necessity			✓		✓	✓	✓	
Avakh Darestani & Hemmati (2019)		✓			–	✓				✓	✓		✓
Hassanpour *et al.* (2019)		✓			–	✓				✓		✓	
Nayeri *et al.* (2020)	✓	✓			–			✓		✓			✓
Atabaki *et al.* (2020)	✓	✓	✓	✓	–		✓					✓	
Abdolazimi *et al.* (2020)		✓			–	✓				✓	✓		✓
Yu & Solvang (2020)	✓	✓	✓	✓	–		✓			✓		✓	
Liu *et al.* (2021)	✓	✓	✓		credibility			✓		✓			✓
Habib *et al.* (2021)	✓	✓	✓		credibility			✓			✓		✓
Lahri *et al.* (2021)	✓		✓		–		✓			✓		✓	
Gilani & Sahebi (2021)	✓	✓	✓		necessity			✓		✓	✓		✓
Foroozesh *et al.* (2022)	✓	✓	✓		credibility			✓		✓	✓		✓
Baghizadeh *et al.* (2022)	✓	✓	✓		*Me*			✓			✓		✓
Ghahremani-Nahr *et al.* (2022)	✓	✓	✓		necessity			✓		✓	✓		✓
Pei *et al.* (2022)	✓	✓			–			✓			✓	✓	
Rouhani & Amin (2022)		✓			–	✓				✓	✓		✓
Ghasemi *et al.* (2022)	✓	✓	✓		necessity			✓			✓		✓
Habib *et al.* (2022)	✓	✓	✓		*Me*			✓			✓		✓
Babaee Tirkolaee *et al.* (2023)	✓		✓		–		✓			✓	✓	✓	
Gao *et al.* (2024)		✓			–	✓					✓	✓	
Ala *et al.* (2024)	✓	✓	✓	✓	–		✓				✓		✓
Guo *et al.* (2024)	✓	✓	✓	✓	–		✓			✓		✓	
This work	✓	✓	✓	✓	possibility; necessity; credibility				✓	✓	✓		✓

3. Model formulation

This section suggests the multi-period and multi-product models considering the hybrid uncertainty for stone paper sustainable CLSCND. The considered SC is a closed-loop in this study and includes MCs, DCs, customer points, CCs, RCs, RSs, and secondary markets. The manufactured products are sent to the customers' points through the DCs in the forward flow. Returned goods from customers are handled in collecting places before being delivered to places for recovery and recycling in the reverse flow. Recovered goods are transported from RCs to DCs for resale. Recycled goods are transmitted to the secondary market. Figure 1 shows the schematic of the proposed SCLSC.

This research proposes a stochastic procedure since many parameters are random, varying values throughout time and scenarios. So, considering the cognitive and stochastic uncertainties based on the views of DMs in this model, the optimal number and location of different facilities are specified for cost minimization and carbon emission goals in the proposed SC. In the considered SC, some assumptions are considered as follows:

- different stone paper products are made based on different applications in MCs;
- the capacity of MCs, DCs, CCs, RCs, and RSs is limited;
- covered cities are fixed for customers' points;
- a certain percentage of returned products from customers' points can be recovered and recycled;
- recovered products are used again in SC;
- recycled products are transferred to the secondary market for sale;
- uncertainty of parameters is considered as PFNs;
- the processing costs, emissions, and demand values are estimated based on the fuzzy scenario.

Optimization model. According to the provided assumptions, the sustainable CLSCND problem is formulated as follows:

$$\min Z_1 = \sum_{i\in\{m,d,l,g,n\}} \widetilde{fc}_i \cdot mc_i + \sum_s P_s \times \Bigg(\sum_q\sum_m\sum_d\sum_t \left(\widetilde{co}_{qmdts} + \tilde{\rho}_{qmts}\right)\cdot o_{qmdts} +$$
$$\sum_q\sum_d\sum_c\sum_t \left(\widetilde{cu}_{qdcts} + \tilde{\pi}_{qdts}\right)\cdot u_{qdcts} +$$
$$\sum_q\sum_c\sum_l\sum_t \widetilde{cq}_{qclts}\cdot q_{qclts} +$$
$$\sum_q\sum_l\sum_g\sum_t \left(\widetilde{cp}_{qlgts} + \tilde{\partial}_{qlts}\right)\cdot p_{qlgts} +$$
$$\sum_q\sum_l\sum_n\sum_t \left(\widetilde{cs}_{qlnts} + \tilde{\partial}_{qlts}\right)\cdot s_{qlnts} +$$
$$\sum_q\sum_g\sum_d\sum_t \left(\widetilde{ch}_{qgdts} + \tilde{\tau}_{qgts}\right)\cdot h_{qgdts} +$$
$$\sum_u\sum_n\sum_v\sum_t \left(\widetilde{cv}_{unvts} + \tilde{\gamma}_{unts}\right)\cdot v_{unvts} \Bigg); \quad (1)$$

$$\min Z_2 = \sum_s P_s \cdot \Bigg(\sum_q\sum_m\sum_d\sum_t \left(\tilde{e}_{qmdts} + \tilde{e}_{qmts}\right)\cdot o_{qmdts} +$$
$$\sum_q\sum_d\sum_c\sum_t \left(\tilde{e}_{qdcts} + \tilde{e}_{qdts}\right)\cdot u_{qdcts} +$$
$$\sum_q\sum_c\sum_l\sum_t \tilde{e}_{qclts}\cdot q_{qclts} +$$
$$\sum_q\sum_l\sum_g\sum_t \left(\tilde{e}_{qlgts} + \tilde{e}_{qlts}\right)\cdot p_{qlgts} +$$
$$\sum_q\sum_l\sum_n\sum_t \left(\tilde{e}_{qlnts} + \tilde{e}_{qlts}\right)\cdot s_{qlnts} +$$
$$\sum_q\sum_g\sum_d\sum_t \left(\tilde{e}_{qgdts} + \tilde{e}_{qgts}\right)\cdot h_{qgdts} +$$
$$\sum_u\sum_n\sum_v\sum_t \left(\tilde{e}_{unvts} + \tilde{e}_{unts}\right)\cdot v_{unvts} \Bigg); \quad (2)$$

$$\sum_d u_{qdcts} \ge \tilde{d}_{qcts}, \ \forall q, c, t, s; \quad (3)$$

$$\sum_n v_{unvts} \ge \tilde{d}_{uvt}, \ \forall u, v, t, s; \quad (4)$$

$$\sum_l q_{qclts} \ge \tilde{r}_{qcts}, \ \forall q, c, t, s; \quad (5)$$

$$\tilde{r}_{qcts} = \widetilde{w}_{qc}\cdot \tilde{d}_{q,c,t-1,s}, \ \forall q, c, t, s; \quad (6)$$

$$\tilde{d}_{q,c,t=0,s} = 0, \ \forall q, c, t, s; \quad (7)$$

$$\sum_m o_{qmdts} + \sum_g h_{qgdts} \ge \sum_k u_{qdkts}, \ \forall q, d, t, s; \quad (8)$$

$$\widetilde{\omega}_{qt}\cdot \sum_c q_{qclts} = \sum_n s_{qlnts}, \ \forall q, l, t, s; \quad (9)$$

$$\left(1-\widetilde{\omega}_{qt}\right)\cdot \sum_c q_{qclts} = \sum_g p_{qlgts}, \ \forall q, l, t, s; \quad (10)$$

$$\sum_d h_{qmdts} = \sum_l p_{qlmts}, \ \forall q, m, t, s; \quad (11)$$

$$\sum_u\sum_v v_{unvts} \le \sum_q\sum_l s_{qlnts}, \ \forall n, t, s; \quad (12)$$

$$\sum_q\sum_d o_{qmdts} \le mc_i \cdot \widetilde{pp}_i, \ \forall m, t, s, \ \forall i \in \{m\}; \quad (13)$$

$$\sum_q\sum_c u_{qdcts} \le mc_i \cdot \widetilde{pp}_i, \ \forall d, t, s, \ \forall i \in \{d\}; \quad (14)$$

$$\sum_q\sum_c q_{qclts} \le mc_i \cdot \widetilde{pp}_i, \ \forall l, t, s, \ \forall i \in \{l\}; \quad (15)$$

$$\sum_q\sum_l p_{qlmts} \le mc_i \cdot \widetilde{pp}_i, \ \forall g, t, s, \ \forall i \in \{g\}; \quad (16)$$

$$\sum_q\sum_l s_{qlnts} \le mc_i \cdot \widetilde{pp}_i, \ \forall n, t, s, \ \forall i \in \{n\}; \quad (17)$$

$$mc_i \in \{0, 1\}, \ \forall i \in \{m, d, l, g, n\}; \quad (18)$$

$$o_{qmdts}, u_{qdcts}, q_{qclts}, p_{qlgts}, s_{qlnts}, h_{qgdts}, v_{unvts} \ge 0,$$
$$\forall q, m, d, c, l, g, n, u, v, t, s. \quad (17)$$

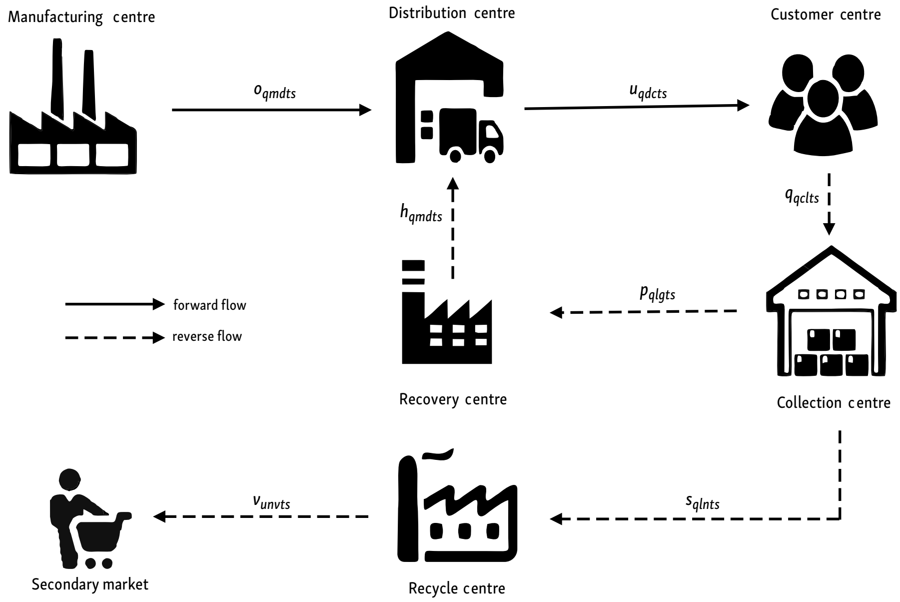

Figure 1. The schematic of the proposed SCLSC

The objective Z_1 deals with minimizing SC costs by minimizing the fixed and variable costs of processing, expressed in Equation (1). The objective Z_2 minimizes total carbon emissions in SCLSC via Equation (2). Equations (3) and (4) refer to the related constraints to the customer's demands and the secondary market. In constraint (5), it is mentioned that the returned goods of all customer places are collected. The number of returned customer goods is measured according to Constraint (6). In time 0, the demand for the good type p in customer place c under scenario s equals zero under Constraint (7). The transfer of products from places of production and recovery to places of distribution is outlined in Constraints (8). Constraint (9) provides the relationship between the transportation of items from CCs to RSs. Transferring goods between collection and recovery places is addressed in Constraint (10). Constraint (11) specifies the relation between the amount of transported goods from RCs to DCs and the amount of transported goods from CCs to RCs. Constraint (12) states the relationship between transmitted goods from CCs to recycling locations and recycled items to secondary markets. Constraints (13)–(17), respectively, refer to the greatest capacity of MCs, DCs, CCs and RCs, and RSs. Constraints (18) and (19) refer to binary and non-negative variables in the model, respectively.

Therefore, based on the proposed CLSCND structure to deal with hybrid uncertainty, a novel RSP programming approach is proposed based on PFN in this research. Criteria for possibility, necessity, and credibility are presented in the proposed approach based on PFN to consider different decision-risk situations. In the following, A linear programming model based on PFNs is presented in the case of stone paper CLSCND to achieve global optimal solutions. The research steps and the proposed approach to consider hybrid uncertainty are presented in Figure 2.

4. The proposed possibilistic programming based on PFNs

In this section, PFNs and their membership function are discussed 1st. Then, a novel possibilistic programming approach is described using PFNs. Possibilistic programming based on PFNs is explained and measured by possibility, necessity, and credibility measures.

4.1. Geometric representation and membership functions of PFNs

Accurate measurement of some data is impossible due to measurement errors and instrument defects. Assume that when measuring temperature and humidity simultaneously, the temperature is roughly 35 °C with ordinary humidity. In this case, the temperature seems to be either more than or less than 35 °C, which impacts the ordinary humidity. It was evident that temperature change also impacts humidity percentage, a common phenomenon (Panda, Pal 2015). A new class of fuzzy numbers known as PFNs is created due to this conceptual diversity. A PFN, as the name suggests, is a subset of a real number R with 5 parameters.

PFNs are used in situations that require more resolution than triangular or trapezoidal numbers. PFNs are capable of more realistic modelling than triangular and trapezoidal numbers. Language variables for triangular fuzzy numbers are low, medium, and high. Trapezoidal numbers have low, low, high, and very high linguistic variables. PFNs have very low, low, medium, high, and very high linguistic variables. The used PFNs were introduced by Sengupta *et al.* (2018). Suppose that the fuzzy number $\widetilde{A}_p$ in Equation (20) is a PFN:

$$\widetilde{A}_p = \left(€^1, €^2, €^3, €^4, €^5\right). \tag{20}$$

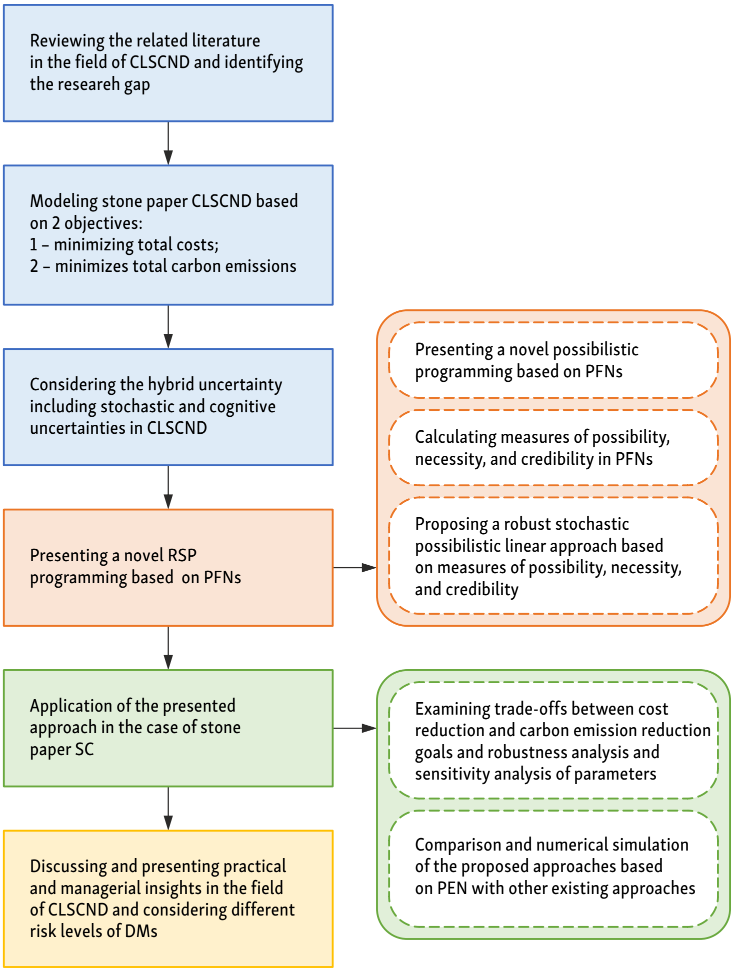

Figure 2. The research steps and the proposed approach to consider hybrid uncertainty in CLSCND

In Equation (20), $€^3$ is the central point and ($€^1$, $€^2$) and ($€^4$, $€^5$) are the left and right scopes of ξ^3.

This part examines PFNs in terms of geometric representation and membership function and compares them with other fuzzy sets. The membership function of the PFNs is illustrated in Figure 3.

ω is the membership degree coefficient of points $€^2$ and $€^4$ in symmetric PFNs. The function of PFNs is defined by Equation (21) as indicated in Figure 3:

$$\mu_{\tilde{A}_p(x)} = \begin{cases} 0, & x \le €^1; \\ \frac{\omega}{€^2 - €^1} \cdot \left(x - €^1\right), & €^1 \le x \le €^2; \\ \omega + \frac{1-\omega}{€^3 - €^2} \cdot \left(x - €^2\right), & €^2 \le x \le €^3; \\ 1, & x = €^3; \\ 1 + \frac{\omega - 1}{€^4 - €^3} \cdot \left(x - €^3\right), & €^3 \le x \le €^4; \\ \omega + \frac{-\omega}{€^5 - €^4} \cdot \left(x - €^4\right), & €^4 \le x \le €^5; \\ 0, & x \ge €^5. \end{cases} \quad (21)$$

Also, the average or expected value of PFNs is calculated according to Equation (21) as follows (Zohrehvandi *et al.* 2020):

$$M\left(\tilde{A}_p\right) = \frac{€^1 + 4 \cdot €^2 + 6 \cdot €^3 + 4 \cdot €^4 + €^5}{16}. \quad (22)$$

Suppose a PFN is defined in a general way, and according to the values of the membership degree coefficient, 2 specific fuzzy numbers, i.e., trapezoidal and triangular fuzzy numbers, are investigated. Based on the values of the membership degree coefficient, 2 states are reviewed:

- *State I*: where $\omega = 0$, then the PFN becomes a triangular fuzzy number, $\tilde{A}_p = \left(€^1, €^2, €^3, €^4, €^5\right) \cong \left(€^2, €^3, €^4\right)$:

$$\mu_{\tilde{A}_p(x)} = \begin{cases} 0, & x \le €^2; \\ \frac{x - €^2}{€^3 - €^2}, & €^2 \le x \le €^3; \\ 1, & x = €^3; \\ 1 + \frac{x - €^3}{€^3 - €^4}, & €^3 \le x \le €^4; \\ 0, & x \ge €; \end{cases} \quad (23)$$

- *State II*: where $\omega = 1$, then the PFN becomes a trapezoidal fuzzy number, $\tilde{A}_p = \left(€^1, €^2, €^3, €^4, €^5\right) \cong \left(€^1, €^2, €^4, €^5\right)$:

$$\mu_{\tilde{A}_p}(x)=\begin{cases}0, & x\le €^1;\\ \frac{1}{€^2-€^1}\cdot\left(x-€^1\right), & €^1\le x\le €^2;\\ 1, & €^2\le x\le €^4;\\ 1+\left(\frac{-1}{€^5-€^4}\right)\cdot\left(x-€^4\right), & €^4\le x\le €^5;\\ 0, & x\ge €^5.\end{cases} \quad (24)$$

Figure 4 compares the functions of fuzzy numbers.

PFNs are more comprehensive than triangular and trapezoidal fuzzy numbers, and they are converted into triangular and trapezoidal fuzzy numbers based on different values of the membership degree coefficient. PFNs can be converted into 2 specific states: triangular and trapezoidal fuzzy numbers. As a result, compared to triangular and trapezoidal fuzzy numbers, PFNs are more accurate and complete when considering uncertainty.

4.2. Possibilistic programming based on PFNs

In actual conditions, optimization problems are faced with uncertainty about many parameters. Fuzzy programming can be divided into possibilistic and flexible (Hosseini Dehshiri *et al.* 2022). A type of fuzzy mathematical programming known as possibilistic programming derives from the probability theory and focuses on ambiguous and fuzzy coefficients of constraints and objective functions.

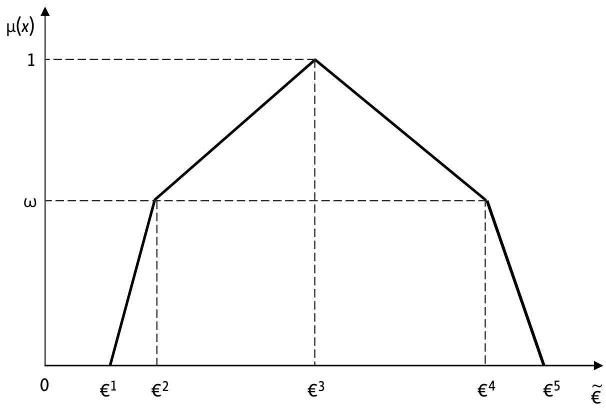

Figure 3. Membership function of fuzzy number $\tilde{€}$

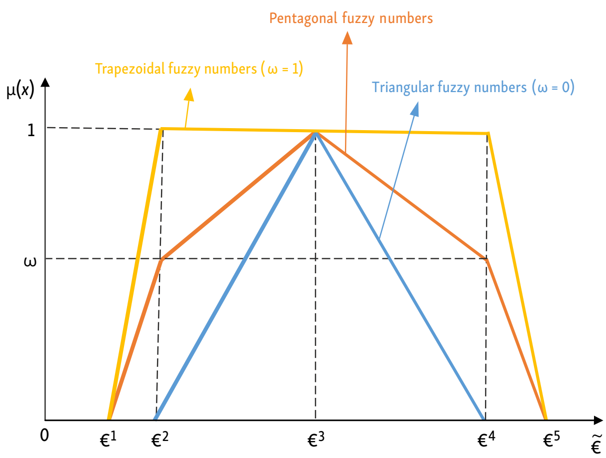

Figure 4. Comparison of functions of fuzzy numbers

Possibilistic programming responds to cognitive uncertainties. In this method, DMs can meet the constraints by using the average value of fuzzy and considering the lowest degrees of confidence. This procedure has 2 types of possibility and necessity measures (Dehghan *et al.* 2018). The necessary criterion is towards the smallest probability of occurrence of the ambiguous parameter or the pessimistic condition; conversely, the probability criterion is towards the maximum probability of occurrence of the ambiguous parameter (Pishvaee *et al.* 2012).

The possibilistic programming approach has been used for triangular and trapezoidal fuzzy numbers. Still, possibilistic programming has not yet been developed for PFNs, as can be seen from the studies done in the field. This is even though PFNs are more comprehensive than triangular and trapezoidal fuzzy sets and can consider the maximum subjectivity of DMs compared to other fuzzy sets. Consequently, the approach of possibilistic programming is developed on the necessity, possibility, and credibility criteria for PFNs.

4.2.1. Calculating possibility measure

The possibility measure considers the optimistic state in evaluating DMs in possibilistic programming. According to Figure 3 and the membership function of PFNs in Equation (20), the possibility measure is calculated based on PFNs. In this regard, surveys are conducted in 6 areas. The membership function of the possibility measure is expressed in Equations (25) and (26):

$$Pos\left(\tilde{A}\le x\right)=\begin{cases}0, & x\le €^1;\\ \frac{\omega\cdot\left(x-€^1\right)}{€^2-€^1}, & €^1\le x\le €^2;\\ \omega+\frac{1-\omega}{€^3-€^2}\cdot\left(x-€^2\right), & €^2\le x\le €^3;\\ 1, & €^3\le x\le €^4;\\ 1, & €^4\le x\le €^5;\\ 1, & x\ge €^5;\end{cases} \quad (25)$$

$$Pos\left(\tilde{A}\ge x\right)=\begin{cases}1, & x\le €^1;\\ 1, & €^1\le x\le €^2;\\ 1, & €^2\le x\le €^3;\\ 1+\frac{\omega-1}{€^4-€^3}\cdot\left(x-€^3\right), & €^3\le x\le €^4;\\ \omega+\frac{-\omega}{€^5-€^4}\cdot\left(x-€^4\right), & €^4\le x\le €^5;\\ 0, & x\ge €^5.\end{cases} \quad (26)$$

The possibility measure values are calculated using Equations (25) and (26). For $0 \le \alpha \le 0.5$, the possibility measure values are calculated as follows:

$$Pos\left(\tilde{A}\le x\right)\ge\alpha\Leftrightarrow$$

$$\left(\frac{\omega\cdot\left(x-€^1\right)}{€^2-€^1}\right)\ge\alpha\Leftrightarrow$$

$$x\ge\frac{€^1\cdot\left(\omega-\alpha\right)+\alpha\cdot €^2}{\omega}; \quad (27)$$

$$Pos\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow$$
$$\omega + \left(\frac{-\omega}{€^5 - €^4}\right) \cdot \left(x - €^4\right) \ge \alpha \Leftrightarrow$$
$$x \le \frac{€^5 \cdot \left(\omega - \alpha\right) + \alpha \cdot €^4}{\omega}. \quad (28)$$

Next, the possibility measure values for $0.5 \le \alpha \le 1$ are calculated as follows:

$$Pos\left(\tilde{A} \le x\right) \ge \alpha \Leftrightarrow$$
$$\omega + \left(\frac{1-\omega}{€^3 - €^2}\right) \cdot \left(x - €^2\right) \ge \alpha \Leftrightarrow$$
$$x \ge \frac{€^3 \cdot \left(\alpha - \omega\right) + €^2 \cdot \left(1-\alpha\right)}{1-\omega}; \quad (29)$$

$$Pos\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow$$
$$1 + \left(\frac{\omega - 1}{€^4 - €^3}\right) \cdot \left(x - €^3\right) \ge \alpha \Leftrightarrow$$
$$x \le \frac{€^3 \cdot \left(\alpha - \omega\right) + €^4 \cdot \left(1-\alpha\right)}{1-\omega}. \quad (30)$$

4.2.2. Calculating necessity measure

The necessity criterion was created to consider the negative viewpoints in possibilistic programming. The function of the necessity standard is as follows, according to Figure 3 and Equation (21):

$$Nec\left(\tilde{A} \le x\right) = \begin{cases} 0, & x \le €^1; \\ 0, & €^1 \le x \le €^2; \\ 0, & €^2 \le x \le €^3; \\ -\dfrac{\omega - 1}{€^4 - €^3} \cdot \left(x - €^3\right), & €^3 \le x \le €^4; \\ 1 - \omega + \dfrac{\omega \cdot \left(x - €^4\right)}{€^5 - €^4)}, & €^4 \le x \le €^5; \\ 1, & x \ge €^5; \end{cases} \quad (31)$$

$$Nec\left(\tilde{A} \ge x\right) = \begin{cases} 1, & x \le €^1; \\ 1 - \dfrac{\omega \cdot \left(x - €^1\right)}{€^2 - €^1}, & €^1 \le x \le €^2; \\ 1 - \omega - \dfrac{1-\omega}{€^3 - €^2} \cdot \left(x - €^2\right), & €^2 \le x \le €^3; \\ 0, & €^3 \le x \le €^4; \\ 0, & €^4 \le x \le €^5; \\ 0, & x \ge €^5. \end{cases} \quad (32)$$

Next, the necessity measure values are calculated using Equations (31) and (32). For $0 \le \alpha \le 0.5$, the necessity measure values are calculated as follows:

$$Nec\left(\tilde{A} \le x\right) \ge \alpha \Leftrightarrow$$
$$-\left(\frac{\omega - 1}{€^4 - €^3}\right) \cdot \left(x - €^3\right) \ge \alpha \Leftrightarrow$$
$$x \ge \frac{€^3 \cdot \left(1 - \omega - \alpha\right) + \alpha \cdot €^4}{1-\omega}; \quad (33)$$

$$Nec\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow$$
$$1 - \omega - \left(\frac{1-\omega}{€^3 - €^2}\right) \cdot \left(x - €^2\right) \ge \alpha \Leftrightarrow$$
$$x \le \frac{€^3 \cdot \left(1 - \omega - \alpha\right) + \alpha \cdot €^2}{1-\omega}. \quad (34)$$

The necessity measure values for $0.5 \le \alpha \le 1$ are calculated as follows:

$$Nec\left(\tilde{A} \le x\right) \ge \alpha \Leftrightarrow$$
$$1 - \omega + \left(\frac{\omega \cdot \left(x - €^4\right)}{€^5 - €^4}\right) \ge \alpha \Leftrightarrow$$
$$x \ge \frac{€^5 \cdot \left(\omega + \alpha - 1\right) + €^4 \cdot \left(1-\alpha\right)}{\omega}; \quad (35)$$

$$Nec\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow$$
$$1 - \left(\frac{\omega \cdot \left(x - €^1\right)}{€^2 - €^1}\right) \ge \alpha \Leftrightarrow$$
$$x \le \frac{€^1 \cdot \left(\omega + \alpha - 1\right) + €^2 \cdot \left(1-\alpha\right)}{\omega}. \quad (36)$$

4.2.3. Calculating credibility measure

According to the literature, credibility is specified by the average of necessity and possibility. Credibility measures are as follows:

$$Cr\left(\tilde{A} \cdot x\right) = \frac{1}{2} \cdot \left(Pos\left(\tilde{A} \cdot x\right) + Nec\left(\tilde{A} \cdot x\right)\right); \quad (37)$$

$$Cr\left(\tilde{A} \le x\right) = \begin{cases} 0, & x \le €^1; \\ \dfrac{1}{2} \cdot \dfrac{\omega \cdot \left(x - €^1\right)}{€^2 - €^1}, & €^1 \le x \le €^2; \\ \dfrac{1}{2} \cdot \left(\omega + \dfrac{1-\omega}{€^3 - €^2} \cdot \left(x - €^2\right)\right), & €^2 \le x \le €^3; \\ \dfrac{1}{2} + \left(1 - \dfrac{1}{2}\right) \cdot \left(-\dfrac{\omega - 1}{€^4 - €^3} \cdot \left(x - €^3\right)\right), & €^3 \le x \le €^4; \\ \dfrac{1}{2} + \left(1 - \dfrac{1}{2}\right) \cdot \left(1 - \omega + \dfrac{\omega \cdot \left(x - €^4\right)}{€^5 - €^4}\right), & €^4 \le x \le €^5; \\ 1, & x \ge €^5; \end{cases} \quad (38)$$

$$Cr\left(\tilde{A} \ge x\right) = \begin{cases} 1, & x \le €^1; \\ \dfrac{1}{2} + \left(1 - \dfrac{1}{2}\right) \cdot \left(1 - \dfrac{\omega \cdot \left(x - €^1\right)}{€^2 - €^1}\right), & €^1 \le x \le €^2; \\ \dfrac{1}{2} + \left(1 - \dfrac{1}{2}\right) \cdot \left(1 - \omega - \dfrac{1-\omega}{€^3 - €^2} \cdot \left(x - €^2\right)\right), & €^2 \le x \le €^3; \\ \dfrac{1}{2} \cdot \left(1 + \dfrac{\omega - 1}{€^4 - €^3} \cdot \left(x - €^3\right)\right), & €^3 \le x \le €^4; \\ \dfrac{1}{2} \cdot \left(\omega + \dfrac{-\omega}{€^5 - €^4} \cdot \left(x - €^4\right)\right), & €^4 \le x \le €^5; \\ 0, & x \ge €^5. \end{cases} \quad (39)$$

Credibility measure values are calculated using Equations (38) and (39). The value of α is examined in 4 intervals to calculate the credibility measure.

For $0 \le \alpha \le 0.25$, credibility measure values are calculated as follows:

$$Cr\left(\tilde{A} \le x\right) \ge \alpha \Leftrightarrow \frac{1}{2}\cdot\left(\frac{\omega\cdot\left(x-€^{1}\right)}{€^{2}-€^{1}}\right) \ge \alpha \Leftrightarrow$$

$$x \ge \frac{€^{1}\cdot\left(\omega-2\cdot\alpha\right)+2\cdot\alpha\cdot €^{2}}{\omega}; \tag{40}$$

$$Cr\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow \frac{1}{2}\cdot\left(\omega+\left(\frac{-\omega}{€^{5}-€^{4}}\right)\cdot\left(x-€^{4}\right)\right) \ge \alpha \Leftrightarrow$$

$$x \le \frac{€^{5}\cdot\left(\omega-2\cdot\alpha\right)+2\cdot\alpha\cdot €^{4}}{\omega}. \tag{41}$$

For $0.25 \le \alpha \le 0.5$, credibility measure values are calculated as follows:

$$Cr\left(\tilde{A} \le x\right) \ge \alpha \Leftrightarrow \frac{1}{2}\cdot\left(\omega+\left(\frac{1-\omega}{€^{3}-€^{2}}\right)\cdot\left(x-€^{2}\right)\right) \ge \alpha \Leftrightarrow$$

$$x \ge \frac{€^{3}\cdot\left(2\cdot\alpha-\omega\right)+€^{2}\cdot\left(1-2\cdot\alpha\right)}{1-\omega}; \tag{42}$$

$$Cr\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow \frac{1}{2}\cdot\left(1+\left(\frac{\omega-1}{€^{4}-€^{3}}\right)\cdot\left(x-€^{3}\right)\right) \ge \alpha \Leftrightarrow$$

$$x \le \frac{€^{3}\cdot\left(2\cdot\alpha-\omega\right)+€^{4}\cdot\left(1-2\cdot\alpha\right)}{1-\omega}. \tag{43}$$

For $0.5 \le \alpha \le 0.75$, credibility measure values are calculated as follows:

$$Cr\left(\tilde{A} \le x\right) \ge \alpha \Leftrightarrow$$

$$\frac{1}{2}+\left(1-\frac{1}{2}\right)\cdot\left(-\left(\frac{\omega-1}{€^{4}-€^{3}}\right)\cdot\left(x-€^{3}\right)\right) \ge \alpha \Leftrightarrow$$

$$x \ge \frac{€^{3}\cdot\left(2-2\cdot\alpha-\omega\right)+€^{4}\cdot\left(2\cdot\alpha-1\right)}{1-\omega}; \tag{44}$$

$$Cr\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow$$

$$\frac{1}{2}+\left(1-\frac{1}{2}\right)\cdot\left(1-\omega-\left(\frac{1-\omega}{€^{3}-€^{2}}\right)\cdot\left(x-€^{2}\right)\right) \ge \alpha \Leftrightarrow$$

$$x \le \frac{€^{3}\cdot\left(2-2\cdot\alpha-\omega\right)+€^{2}\cdot\left(2\cdot\alpha-1\right)}{1-\omega}. \tag{45}$$

For $0.75 \le \alpha \le 1$ credibility measure values are calculated as follows:

$$Cr\left(\tilde{A} \le x\right) \ge \alpha \Leftrightarrow$$

$$\frac{1}{2}+\left(1-\frac{1}{2}\right)\cdot\left(1-\omega+\left(\frac{\omega\cdot\left(x-€^{4}\right)}{€^{5}-€^{4}}\right)\right) \ge \alpha \Leftrightarrow$$

$$x \ge \frac{€^{5}\cdot\left(2\cdot\alpha+\omega-2\right)+€^{4}\cdot\left(2-2\cdot\alpha\right)}{\omega}; \tag{46}$$

$$Cr\left(\tilde{A} \ge x\right) \ge \alpha \Leftrightarrow$$

$$\frac{1}{2}+\left(1-\frac{1}{2}\right)\cdot\left(1-\frac{\omega\cdot\left(x-€^{1}\right)}{€^{2}-€^{1}}\right) \ge \alpha \Leftrightarrow$$

$$x \le \frac{€^{1}\cdot\left(2\cdot\alpha+\omega-2\right)+€^{2}\cdot\left(2-2\cdot\alpha\right)}{\omega}. \tag{47}$$

5. The novel RSP programming based on PFNs

Possibilistic programming has weaknesses such that using possibilistic programming leads to ignoring deviations, and decisions are typically made under moderate situations. Also, the minimum confidence level in possibilistic programming is not optimally determined and requires repeated examinations (Hosseini Dehshiri *et al.* 2022). In addition, in many SCLSC problems, some parameters have random uncertainty, with different values under different scenarios. Stochastic programming should be used in this situation (Farrokh *et al.* 2018). Therefore, the combined approach of the stochastic-possibilistic procedure is recommended to handle cognitive and random uncertainties.

On the other hand, in modelling SCLSC problems, unknown and uncertain parameters should be robust; Otherwise, the effect of parameter fluctuations will be excessive over time (Pishvaee *et al.* 2011). Robust programming was created to address this flaw in SPP. To address uncertainty difficulties, robust programming or risk-averse method optimization theory was developed (Pishvaee *et al.* 2012). If the solution has both feasibility and optimality, the solution is robust. The research literature suggests using robust optimization to address stochastic-possibilistic approach weaknesses (Dehghan *et al.* 2018; Hosseini Dehshiri *et al.* 2022). Based on the weaknesses above, we developed a novel stochastic probabilistic programming framework based on PFNs in this study. The presented procedure controls the possibility and scenario deviations, and the confidence level is determined optimally. The model also provides robust solutions not sensitive to parameter changes regarding feasibility and optimality. The compact form of SCLSC modelling can be expressed as follows:

$$\min E = \tilde{f}\cdot y+\tilde{c}_{s}\cdot x_{s}$$

subject to:

$$A\cdot x_{s} \ge \tilde{d}_{s};$$
$$B\cdot x = 0;$$
$$S\cdot x_{s} \le \tilde{N}_{s}\cdot y;$$
$$T\cdot y \le 1;$$
$$y \in \{0,1\};$$
$$x \ge 0. \tag{48}$$

The vectors f and c in Equation (48) determine the fixed and variable costs. *N* indicates the facility's maximum capacity, *A*, *B*, *T*, and *S* indicate the constraints' coefficients, and d is demand. The variables *x* and *y* are categorized as continuous and binary, respectively. For the SCLSC problem, the coefficient matrix *N* and the vectors *f*, *c*, and *d* represent uncertain parameters in PFNs. The s index indicates the random uncertainty in modelling. In the possibilistic programming approach, the expected value operator and the calculation of mathematical expectation are used for the crisp equivalent of the objective function (Hosseini Dehshiri *et al.* 2022). Additionally, possibility, necessity, and credibility criteria address chance constraints, including uncertain parameters.

5.1. RSP programming model based on possibility measure

This section defines the RSP model based on possibility measures. The compact form of the possibility measure of the SCLSC problem is the following:

$$\min E(z) = E(\tilde{f}) \cdot y + E(\tilde{c}_s) \cdot x_s$$

subject to:

$$\begin{aligned} & Pos\{A \cdot x_s \geq \tilde{d}_s\} \geq \alpha; \\ & B \cdot x = 0; \\ & Pos\{S \cdot x_s \leq \widetilde{N}_s \cdot y\} \geq \beta; \\ & T \cdot y \leq 1; \\ & y \in \{0, 1\}; \\ & x \geq 0. \end{aligned} \tag{49}$$

The objective function is converted into a deterministic state using the mathematical expectation operator by averaging the 1st and 2nd terms. Here is the mathematical expectation of PFNs' objective function:

$$E(z) = \left(\frac{f^1 + 4 \cdot f^2 + 6 \cdot f^3 + 4 \cdot f^4 + f^5}{16}\right) \cdot y + \sum_s P_s \cdot \left(\frac{c^1 + 4 \cdot c^2 + 6 \cdot c^3 + 4 \cdot c^4 + c^5}{16}\right) \cdot x_s. \tag{50}$$

The robust stochastic probabilistic programming model based on possibility measure is investigated in 2 cases:

- *Case I*: for 0 ≤ α, β ≤ 0.5, the RSP programming model is formulated for PFNs as follows according to Equations (27), (28), and (49):

$$\min E(z) + ¥ \cdot (z_{max} - z_{min}) + \Phi \cdot \sum_s P_s \cdot |E(z) - E(z_s)| + \theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^1 \cdot (\omega - \alpha) + \alpha \cdot d_s^2}{\omega}\right)\right) + \pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^5 \cdot (\omega - \beta) + \beta \cdot N_s^4}{\omega}\right) - N_s^1\right) \cdot y$$

subject to:

$$\begin{aligned} & A \cdot x_s \geq \frac{d_s^1 \cdot (\omega - \alpha) + \alpha \cdot d_s^2}{\omega}; \\ & B \cdot x = 0; \\ & S \cdot x_s \leq \frac{N_s^5 \cdot (\omega - \beta) + \beta \cdot N_s^4}{\omega} \cdot y; \\ & T \cdot y \leq 1; \\ & 0 \leq \alpha, \beta \leq 0.5; \\ & y \in \{0, 1\}; \\ & x \geq 0. \end{aligned} \tag{51}$$

The 1st term reduces the mean, while the 2nd term regulates optimality by decreasing the difference between the maximum and minimum. In the 2nd term, $z_{\max} = f_5 \cdot y + c_5 \cdot x$, and $z_{\min} = f_1 \cdot y + c_1 \cdot x$. The coefficient ¥ determines the importance or weight of the 2nd term. The scenario deviation is calculated in the 3rd term, and the coefficient Φ determines the importance or weight of the 3rd term. Feasibility robustness is guaranteed in the 4th and 5th terms.

Since the Model (51) is non-linear, changing the variable $(\vartheta_{p1} = \beta \cdot y)$ and using the methodology described by Yu & Li (2000) can alter it into a linear model. Therefore, the linear model of the RSP approach for the possibility measure is as follows for 0 ≤ α, β ≤ 0.5:

$$\min E(z) + ¥ \cdot (z_{max} - z_{min}) + \Phi \cdot \sum_s P_s \cdot \left((E(z) - E(z_s)) + 2 \cdot \varepsilon_s\right) + \theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^1 \cdot (\omega - \alpha) + \alpha \cdot d_s{}^2}{\omega}\right)\right) + \pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^5 \cdot (\omega \cdot y - \vartheta_{p1}) + N_s^4 \cdot \vartheta_{p1}}{\omega}\right) - N_s^1 \cdot y\right)$$

subject to:

$$\begin{aligned} & E(z) - E(z_s) + \varepsilon_s \geq 0, \ \forall s; \\ & A \cdot x_s \geq \frac{d_s^1 \cdot (\omega - \alpha) + \alpha \cdot d_s^2}{\omega}; \\ & B \cdot x = 0; \\ & S \cdot x_s \leq \frac{N_s^5 \cdot (\omega \cdot y - \vartheta_{p1}) + N_s^4 \cdot \vartheta_{p1}}{\omega}; \\ & \vartheta_{p1} \leq M \cdot y; \\ & \vartheta_{p1} \geq M \cdot (y - 1) + \beta; \\ & \vartheta_{p1} \leq \beta; \\ & \vartheta_{p1} \geq 0; \\ & T \cdot y \leq 1' \\ & 0 \leq \alpha, \beta \leq 0.5; \\ & y \in \{0, 1\}; \\ & x, \varepsilon_s \geq 0. \end{aligned} \tag{52}$$

- *Case II*: for 0.5 ≤ α, β ≤ 1, the RSP programming model is formulated for PFNs as follows according to Equations (29), (30), and (49):

$$\min E(z) + ¥ \cdot (z_{\max} - z_{\min}) + \Phi \cdot \sum_s P_s \cdot |E(z) - E(z_s)| + \theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^3 \cdot (\alpha - \omega) + d_s^2 \cdot (1 - \alpha)}{1 - \omega}\right)\right) + \pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 \cdot (\beta - \omega) + N_s^4 \cdot (1 - \beta)}{1 - \omega}\right) - N_s{}^1\right) \cdot y$$

subject to:

$$A \cdot x_s \geq \frac{d_s^3 \cdot (\alpha - \omega) + d_s^2 \cdot (1 - \alpha)}{1 - \omega};$$

$$B \cdot x = 0;$$
$$S \cdot x_s \le \frac{N_s^3 \cdot (\beta - \omega) + N_s^4 \cdot (1-\beta)}{1-\omega} \cdot y;$$
$$T \cdot y \le 1;$$
$$0.5 \le \alpha;\ \beta \le 1;$$
$$y \in \{0, 1\};$$
$$x \ge 0. \quad (53)$$

The Model (53) can be linearized by altering the variable $\vartheta_{p2} = \beta \cdot y$. According to the expressed method, the absolute value will be eliminated, and the model becomes linear. The linear model of the RSP approach for the possibility measure is as follows for 0 ≤ α, β ≤ 0.5:

$$\min E(z) + ¥ \cdot (z_{max} - z_{min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left(\left(E(z) - E(z_s) \right) + 2 \cdot \varepsilon_s \right) +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^3 \cdot (\alpha - \omega) + d_s^2 \cdot (1-\alpha)}{1-\omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 \cdot (\vartheta_{p2} - \omega \cdot y) + N_s^4 \cdot (y - \vartheta_{p2})}{1-\omega} \right) - N_s^1 \cdot y \right)$$

subject to:

$$E(z) - E(z_s) + \varepsilon_s \ge 0,\ \forall s;$$
$$A \cdot x_s \ge \frac{d_s^3 \cdot (\alpha - \omega) + d_s^2 \cdot (1-\alpha)}{1-\omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \le \frac{N_s^3 \cdot (\vartheta_{p2} - \omega \cdot y) + N_s^4 \cdot (y - \vartheta_{p2})}{1-\omega};$$
$$\vartheta_{p2} \le M \cdot y;$$
$$\vartheta_{p2} \ge M \cdot (y-1) + \beta;$$
$$\vartheta_{p2} \le \beta;$$
$$\vartheta_{p2} \ge 0;$$
$$T \cdot y \le 1;$$
$$0.5 \le \alpha,\ \beta \le 1;$$
$$y \in \{0, 1\};$$
$$x, \varepsilon_s \ge 0. \quad (54)$$

Equations (52) and (54) are the final linear models of the RSP programming of PFNs based on the possibility measure for 0 ≤ α, β ≤ 0.5, and 0.5 ≤ α, β ≤ 1, respectively.

5.2. RSP programming model based on necessity measure

The RSP approach based on necessity is described in this section. The compact form of the necessity measure of the SCLSC problem is the following:

$$\min E(z) = E(\tilde{f}) \cdot y + E(\tilde{c}_s) \cdot x_s$$

subject to:

$$Nec\{A \cdot x_s \ge \tilde{d}_s\} \ge \alpha;$$
$$B \cdot x = 0;$$
$$Nec\{S \cdot x_s \le \widetilde{N}_s \cdot y\} \ge \beta;$$
$$T \cdot y \le 1;$$
$$y \in \{0, 1\};$$
$$x \ge 0. \quad (55)$$

2 cases are considered to examine the RSP programming model based on the necessity measure:

- *Case I*: for 0 ≤ α, β ≤ 0.5, the RSP programming of PFNs is formulated based on PFNs as follows according to Equations (33), (34), and (55):

$$\min E(z) + ¥ \cdot (z_{max} - z_{min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left| E(z) - E(z_s) \right| +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^3 \cdot (1 - \omega - \alpha) + \alpha \cdot d_s^4}{1-\omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 \cdot (1 - \omega - \beta) + \beta \cdot N_s^2}{1-\omega} \right) - N_s^1 \right) \cdot y$$

subject to:

$$A \cdot x_s \ge \frac{d_s^3 \cdot (1 - \omega - \alpha) + \alpha \cdot d_s^4}{1-\omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \le \left(\frac{N_s^3 \cdot (1 - \omega - \beta) + \beta \cdot N_s^2}{1-\omega} \right) \cdot y;$$
$$T \cdot y \le 1;$$
$$0 \le \alpha,\ \beta \le 0.5;$$
$$y \in \{0, 1\};$$
$$x \ge 0. \quad (56)$$

The Model (56) is non-linear and by variable changing $\vartheta_{n1} = \beta \cdot y$, it was converted into a linear model. According to the expressed method, the absolute value will be eliminated, and the model becomes linear. Therefore, the linear model of the RSP programming is as follows for necessity measure and 0 ≤ α, β ≤ 0.5:

$$\min E(z) + ¥ \cdot (z_{max} - z_{min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left(\left(E(z) - E(z_s) \right) + 2 \cdot \varepsilon_s \right) +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^3 \cdot (1 - \omega - \alpha) + \alpha \cdot d_s^4}{1-\omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 \cdot (y - \omega \cdot y - \vartheta_{n1}) + \vartheta_{n1} \cdot N_s^2}{1-\omega} \right) - N_s^1 \cdot y \right)$$

subject to:

$$E(z) - E(z_s) + \varepsilon_s \ge 0,\ \forall s;$$
$$A \cdot x_s \ge \frac{d_s^3 \cdot (1 - \omega - \alpha) + \alpha \cdot d_s^4}{1-\omega};$$

$$B\cdot x=0;$$
$$S\cdot x_s\le\frac{N_s^3\cdot\left(y-\omega\cdot y-\vartheta_{n1}\right)+\vartheta_{n1}\cdot N_s^2}{1-\omega};$$
$$\vartheta_{n1}\le M\cdot y;$$
$$\vartheta_{n1}\ge M\cdot\left(y-1\right)+\beta;$$
$$\vartheta_{n1}\le\beta;$$
$$\vartheta_{n1}\ge 0;$$
$$T\cdot y\le 1$$
$$0\le\alpha,\ \beta\le 0.5;$$
$$y\in\{0,1\};$$
$$x,\varepsilon_s\ge 0. \qquad (57)$$

- *Case II*: for 0.5 ≤ α, β ≤ 1, the RSP programming is formulated as follows based on PFNs according to Equations (35), (36), and (55):

$$\min E(z)+¥\cdot\left(z_{\max}-z_{\min}\right)+$$
$$\Phi\cdot\sum_s P_s\cdot\left|E(z)-E(z_s)\right|+$$
$$\theta\cdot\sum_s P_s\cdot\left(d_s^{\,5}-\left(\frac{d_s^5\cdot(\omega+\alpha-1)+d_s^4\cdot(1-\alpha)}{\omega}\right)\right)+$$
$$\pi\cdot\sum_s P_s\cdot\left(\left(\frac{N_s^1\cdot(\omega+\beta-1)+N_s^2\cdot(1-\beta)}{\omega}\right)-N_s^1\right)\cdot y$$

subject to:

$$A\cdot x_s\ge\frac{d_s^5\cdot(\omega+\alpha-1)+d_s^4\cdot(1-\alpha)}{\omega};$$
$$B\cdot x=0;$$
$$S\cdot x_s\le\left(\frac{N_s^1\cdot(\omega+\beta-1)+N_s^2\cdot(1-\beta)}{\omega}\right)\cdot y;$$
$$T\cdot y\le 1;$$
$$0.5\le\alpha,\ \beta\le 1;$$
$$y\in\{0,1\};$$
$$x\ge 0. \qquad (58)$$

Equation (58) is made into a linear model by altering the variable $\vartheta_{n2}=\beta\cdot y$, and in the proposed linear model, M is a large number. The linear form of RSP programming is as follows for necessity measure and 0.5 ≤ α, β ≤ 1:

$$\min E(z)+¥\cdot\left(z_{\max}-z_{\min}\right)+$$
$$\Phi\cdot\sum_s P_s\cdot\left(\left(E(z)-E(z_s)\right)+2\cdot\varepsilon_s\right)+$$
$$\theta\cdot\sum_s P_s\cdot\left(d_s^{\,5}-\left(\frac{d_s^5\cdot(\omega+\alpha-1)+d_s^4\cdot(1-\alpha)}{\omega}\right)\right)+$$
$$\pi\cdot\sum_s P_s\cdot\left(\left(\frac{N_s^1\cdot\left(\omega\cdot y+\vartheta_{n2}-y\right)+N_s^2\cdot\left(y-\vartheta_{n2}\right)}{\omega}\right)-N_s^1\cdot y\right)$$

subject to:

$$E(z)-E(z_s)+\varepsilon_s\ge 0,\ \forall s;$$
$$A\cdot x_s\ge\frac{d_s^5\cdot(\omega+\alpha-1)+d_s^4\cdot(1-\alpha)}{\omega};$$
$$B\cdot x=0;$$
$$S\cdot x_s\le\frac{N_s^1\cdot\left(\omega\cdot y+\vartheta_{n2}-y\right)+N_s^2\cdot\left(y-\vartheta_{n2}\right)}{\omega};$$
$$\vartheta_{n2}\le M\cdot y;$$
$$\vartheta_{n2}\ge M\cdot(y-1)+\beta;$$
$$\vartheta_{n2}\le\beta;$$
$$\vartheta_{n2}\ge 0;$$
$$T\cdot y\le 1;$$
$$0.5\le\alpha,\ \beta\le 1;$$
$$y\in\{0,1\};$$
$$x,\varepsilon_s\ge 0. \qquad (59)$$

Therefore, according to the presented models, Equations (57) and (59) are, respectively, the final linear models of the RSP programming of PFNs based on the necessity criterion for 0 ≤ α, β ≤ 0.5 and 0.5 ≤ α, β ≤ 1.

5.3. RSP programming model based on credibility measure

This section describes the RSP programming approach based on the credibility measure. The compact form of the credibility measure of the SCLSC problem is the following:

$$\min E(z)=E\left(\tilde f\right)\cdot y+E\left(\tilde c_s\right)\cdot x_s$$

subject to:

$$Cr\left\{Ax_s\ge\tilde d_s\right\}\ge\alpha;$$
$$B\cdot x=0;$$
$$Cr\left\{S\cdot x_s\le\tilde N_s\cdot y\right\}\ge\beta;$$
$$T\cdot y\le 1;$$
$$y\in\{0,1\};$$
$$x\ge 0. \qquad (60)$$

4 cases are considered to calculate the RSP programming model based on credibility measures:

- *Case I*: for 0 ≤ α ≤ 0.25, the RSP programming is formulated based on PFNs as follows according to Equations (40), (41), and (60):

$$\min E(z)+¥\cdot\left(z_{\max}-z_{\min}\right)+$$
$$\Phi\cdot\sum_s P_s\cdot\left|E(z)-E(z_s)\right|+$$
$$\theta\cdot\sum_s P_s\cdot\left(d_s^5-\left(\frac{d_s^1\cdot(\omega-2\cdot\alpha)+2\cdot\alpha\cdot d_s^2}{\omega}\right)\right)+$$
$$\pi\cdot\sum_s P_s\cdot\left(\left(\frac{N_s^2\cdot(\omega-2\cdot\beta)+2\cdot\beta\cdot N_s^4}{\omega}\right)-N_s^1\right)\cdot y$$

subject to:

$$A\cdot x_s\ge\frac{d_s^1\cdot(\omega-2\cdot\alpha)+2\cdot\alpha\cdot d_s^2}{\omega};$$

$$B \cdot x = 0;$$
$$S \cdot x_s \le \frac{N_s^5 \cdot (\omega - 2 \cdot \beta) + 2 \cdot \beta \cdot N_s^4}{\omega} \cdot y;$$
$$T \cdot y \le 1;$$
$$0 \le \alpha,\ \beta \le 0.25;$$
$$y \in \{0, 1\};$$
$$x \ge 0. \quad (61)$$

The model (61) is non-linear and by variable changing $\vartheta_{c1} = \beta \cdot y$, and using the expressed procedure, it can be converted into a linear model. Therefore, the linear model of the RSP programming is as follows based on credibility measure for 0 ≤ α, β ≤ 0.25:

$$\min E(z) + ¥(z_{\max} - z_{\min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left(\left(E(z) - E(z_s) \right) + 2 \cdot \varepsilon_s \right) +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^1 \cdot (\omega - 2 \cdot \alpha) + 2 \cdot \alpha \cdot d_s^2}{\omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^5 \cdot (\omega \cdot y - 2 \cdot \vartheta_{c1}) + 2 \cdot \vartheta_{c1} \cdot N_s^4}{\omega} \right) - N_s^1 \cdot y \right)$$

subject to:

$$E(z) - E(z_s) + \varepsilon_s \ge 0,\ \forall s;$$
$$A \cdot x_s \ge \frac{d_s^1 \cdot (\omega - 2 \cdot \alpha) + 2 \cdot \alpha \cdot d_s^2}{\omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \le \frac{N_s^5 \cdot (\omega \cdot y - 2 \cdot \vartheta_{c1}) + 2 \cdot \vartheta_{c1} \cdot N_s^4}{\omega};$$
$$\vartheta_{c1} \le M \cdot y;$$
$$\vartheta_{c1} \ge M \cdot (y - 1) + \beta;$$
$$\vartheta_{c1} \le \beta;$$
$$\vartheta_{c1} \ge 0;$$
$$T \cdot y \le 1$$
$$0 \le \alpha,\ \beta \le 0.25;$$
$$y \in \{0, 1\};$$
$$x, \varepsilon_s \ge 0; \quad (62)$$

- *Case II*: for 0.25 ≤ α, β ≤ 0.5, the RSP programming is formulated as follows according to Equations (42), (43), and (60):

$$\min E(z) + ¥ \cdot (z_{\max} - z_{\min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left| E(z) - E(z_s) \right| +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^3 \cdot (2 \cdot \alpha - \omega) + d_s^2 \cdot (1 - 2 \cdot \alpha)}{1 - \omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 \cdot (2 \cdot \beta - \omega) + N_s^4 \cdot (1 - 2 \cdot \beta)}{1 - \omega} \right) - N_s^1 \right) \cdot y$$

subject to:

$$A \cdot x_s \ge \frac{d_s^3 \cdot (2 \cdot \alpha - \omega) + d_s^2 \cdot (1 - 2 \cdot \alpha)}{1 - \omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \le \frac{N_s^3 \cdot (2 \cdot \beta - \omega) + N_s^4 \cdot (1 - 2 \cdot \beta)}{1 - \omega} \cdot y;$$
$$T \cdot y \le 1;$$
$$0.25 \le \alpha,\ \beta \le 0.5;$$
$$y \in \{0, 1\};$$
$$x \ge 0. \quad (63)$$

Considering that the Model (63) is non-linear, by changing the variable $\vartheta_{c2} = \beta \cdot y$, the model can be made linear, and the absolute value is converted into a linear model according to the expressed approach. Therefore, the linear model of the RSP programming is as follows based on the credibility measure for 0.25 ≤ α, β ≤ 0.5:

$$\min E(z) + ¥ \cdot (z_{\max} - z_{\min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left(\left(E(z) - E(z_s) \right) + 2 \cdot \varepsilon_s \right) +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s{}^5 - \left(\frac{d_s^3 \cdot (2 \cdot \alpha - \omega) + d_s^2 \cdot (1 - 2 \cdot \alpha)}{1 - \omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 \cdot (2 \cdot \vartheta_{c2} - \omega \cdot y) + N_s^4 \cdot (y - 2 \cdot \vartheta_{c2})}{1 - \omega} \right) - N_s^1 \cdot y \right)$$

subject to:

$$E(z) - E(z_s) + \varepsilon_s \ge 0,\ \forall s;$$
$$A \cdot x_s \ge \frac{d_s^3 \cdot (2 \cdot \alpha - \omega) + d_s^2 \cdot (1 - 2 \cdot \alpha)}{1 - \omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \le \frac{N_s^3 \cdot (2 \cdot \vartheta_{c2} - \omega \cdot y) + N_s^4 \cdot (y - 2 \cdot \vartheta_{c2})}{1 - \omega};$$
$$\vartheta_{c2} \le M \cdot y;$$
$$\vartheta_{c2} \ge M \cdot (y - 1) + \beta;$$
$$\vartheta_{c2} \le \beta;$$
$$\vartheta_{c2} \ge 0$$
$$T \cdot y \le 1$$
$$0.25 \le \alpha,\ \beta \le 0.5;$$
$$y \in \{0, 1\};$$
$$x, \varepsilon_s \ge 0; \quad (64)$$

- *Case III*: for 0.5 ≤ α, β ≤ 0.75, the RSP programming is formulated as follows according to Equations (44), (45), and (60):

$$\min E(z) + ¥ \cdot (z_{\max} - z_{\min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left| E(z) - E(z_s) \right| +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^3 \cdot (2 - 2 \cdot \alpha - \omega) + d_s^4 \cdot (2 \cdot \alpha - 1)}{1 - \omega} \right) \right) +$$

$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 \cdot (2-2\cdot\beta-\omega) + N_s^2 \cdot (2\cdot\beta-1)}{1-\omega} \right) - N_s^1 \right) \cdot y$$

subject to:

$$A \cdot x_s \geq \frac{d_s^3 \cdot (2-2\cdot\alpha-\omega) + d_s^4 \cdot (2\cdot\alpha-1)}{1-\omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \leq \frac{N_s^3 \cdot (2-2\cdot\beta-\omega) + N_s^2 \cdot (2\cdot\beta-1)}{1-\omega} \cdot y;$$
$$T \cdot y \leq 1;$$
$$0.5 \leq \alpha,\ \beta \leq 0.75;$$
$$y \in \{0, 1\};$$
$$x \geq 0. \quad (65)$$

The variable change $\vartheta_{c3} = \beta \cdot y$ is defined to linearize the Model (65), and the absolute value is converted into a linear model according to the expressed approach. Therefore, the linear model of the RSP programming is as follows for 0.5 ≤ α, β ≤ 0.75:

$$\min E(z) + ¥ \cdot (z_{\max} - z_{\min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left(\left(E(z) - E(z_s) \right) + 2 \cdot \varepsilon_s \right) +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^3 \cdot (2-2\cdot\alpha-\omega) + d_s^4 \cdot (2\cdot\alpha-1)}{1-\omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^3 (2\cdot y - 2\cdot\vartheta_{c3} - \omega\cdot y) + N_s^2 \cdot (2\cdot\vartheta_{c3} - y)}{1-\omega} \right) - N_s^1 \cdot y \right)$$

subject to:

$$E(z) - E(z_s) + \varepsilon_s \geq 0,\ \forall s;$$
$$A \cdot x_s \geq \frac{d_s^3 \cdot (2-2\cdot\alpha-\omega) + d_s^4 \cdot (2\cdot\alpha-1)}{1-\omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \leq \frac{N_s^3 \cdot (2\cdot y - 2\cdot\vartheta_{c3} - \omega\cdot y) + N_s^2 \cdot (2\cdot\vartheta_{c3} - y)}{1-\omega};$$
$$\vartheta_{c3} \leq M \cdot y;$$
$$\vartheta_{c3} \geq M \cdot (y-1) + \beta;$$
$$\vartheta_{c3} \leq \beta;$$
$$\vartheta_{c3} \geq 0;$$
$$T \cdot y \leq 1;$$
$$0.5 \leq \alpha,\ \beta \leq 0.75;$$
$$y \in \{0, 1\};$$
$$x, \varepsilon_s \geq 0; \quad (66)$$

- *Case IV*: for 0.75 ≤ α, β ≤ 1, the RSP programming is formulated as follows according to Equations (46), (47), and (60):

$$\min E(z) + ¥ \cdot (z_{\max} - z_{\min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left| E(z) - E(z_s) \right| +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^5 (2\cdot\alpha+\omega-2) + d_s^4 \cdot (2-2\cdot\alpha)}{\omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^1 \cdot (2\cdot\beta+\omega-2) + N_s^2 \cdot (2-2\cdot\beta)}{\omega} \right) - N_s^1 \right) \cdot y$$

subject to:

$$A \cdot x_s \geq \frac{d_s^5 \cdot (2\cdot\alpha+\omega-2) + d_s^4 \cdot (2-2\cdot\alpha)}{\omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \leq \frac{N_s^1 \cdot (2\cdot\beta+\omega-2) + N_s^2 \cdot (2-2\cdot\beta)}{\omega} \cdot y;$$
$$T \cdot y \leq 1;$$
$$0.75 \leq \alpha,\ \beta \leq 1;$$
$$y \in \{0, 1\};$$
$$x \geq 0. \quad (67)$$

In the following, to linearize the Model (67), the change of variable $\vartheta_{c4} = \beta \cdot y$ is used. Then, the absolute value is converted into a linear model. Therefore, the linear model of the RSP programming is as follows for 0.75 ≤ α, β ≤ 1:

$$\min E(z) + ¥ \cdot (z_{\max} - z_{\min}) +$$
$$\Phi \cdot \sum_s P_s \cdot \left(\left(E(z) - E(z_s) \right) + 2 \cdot \varepsilon_s \right) +$$
$$\theta \cdot \sum_s P_s \cdot \left(d_s^5 - \left(\frac{d_s^5 (2\alpha+\omega-2) + d_s^4 (2-2\alpha)}{\omega} \right) \right) +$$
$$\pi \cdot \sum_s P_s \cdot \left(\left(\frac{N_s^1 \cdot (2\cdot\vartheta_{c4} + \omega\cdot y - 2\cdot y) + N_s^2 \cdot (2\cdot y - 2\cdot\vartheta_{c4})}{\omega} \right) - N_s^1 \cdot y \right)$$

subject to:

$$E(z) - E(z_s) + \varepsilon_s \geq 0,\ \forall s;$$
$$A \cdot x_s \geq \frac{d_s^5 \cdot (2\cdot\alpha+\omega-2) + d_s^4 \cdot (2-2\cdot\alpha)}{\omega};$$
$$B \cdot x = 0;$$
$$S \cdot x_s \leq \frac{N_s^1 \cdot (2\cdot\vartheta_{c4} + \omega\cdot y - 2\cdot y) + N_s^2 \cdot (2\cdot y - 2\cdot\vartheta_{c4})}{\omega};$$
$$\vartheta_{c4} \leq M \cdot y;$$
$$\vartheta_{c4} \geq M \cdot (y-1) + \beta;$$
$$\vartheta_{c4} \leq \beta;$$
$$\vartheta_{c4} \geq 0$$
$$T \cdot y \leq 1;$$
$$0.75 \leq \alpha,\ \beta \leq 1;$$
$$y \in \{0, 1\};$$
$$x, \varepsilon_s \geq 0. \quad (68)$$

Therefore, according to the presented models, Equations (62), (64), (66), and (68) are, respectively, the final linear models of the RSP programming based on the credibility measure in the intervals of 0 ≤ α, β ≤ 0.25; 0.25 ≤ α, β ≤ 0.5; 0.5 ≤ α, β ≤ 0.75 and 0.75 ≤ α, β ≤ 1.

6. Implementation and computational results

6.1. Case study description

The presented procedure is examined for the SCLSC issue in a real case to produce stone paper in Iran. The potential for stone paper production is high due to iron ore resources in Iran. Additionally, water is not needed during stone paper production, making it ideal for regions like Iran that lack access to clean water. The production process of cellulose paper leads to environmental pollution and water resources as well as high water consumption, but in the production process of this type of paper, there is no need to use water, and environmental pollution has been minimized. In addition, the high quality of printing on stone paper has made this valuable paper in all printing matters, such as book printing, promotional gifts, and carton making. Therefore, stone paper is a suitable substitute for cellulose paper. Therefore, considering the advantages of stone paper, including many applications, as well as no need for water consumption and no harmful environmental effects, the development of stone paper SC is approved in Iran.

In the proposed SC in this study, 3 types of goods are considered. Forward SC includes MCs, DCs, and final customer points. Reverse SC includes CCs, RCs, RSs, and secondary markets. In forward flow, manufacturing plants are considered in 2 locations that produce the goods. Then, the manufactured goods are sent from the factories to the DCs, which are considered in 8 locations to cover customer points, and goods are shipped to customers in eleven locations in the target markets. Returned customer goods are collected in CCs in 5 locations in the reverse flow. Returned goods are processed in CCs and are divided into 2 levels: recoverable goods and recyclable goods based on quality. Recoverable goods are sent from CCs to RCs in 2 locations. Then, the recovered goods are sent from RCs to DCs for sale to customers. Recycled goods are transferred from CCs to RSs in 2 locations. The materials are sent to the secondary market, which consists of 2 centres. Therefore, in the proposed SC, due to the simultaneous consideration of forward and reverse flow while achieving economic benefits, environmental damage is minimized, and recycled items are used again.

6.2. Input parameters

The proposed SCLSC seeks to meet customer and secondary market demand by reducing costs and carbon emissions. The capacity of MCs, DCs, CCs, RCs, and RSs is responsive to demand. 2 consumption periods in a year are considered for demand changes. As a result, the problem is a multi-product and multi-period SCLSC. This study considers 4 states for random parameters to deal with random uncertainty. Similarly, low, normal, high, and very high conditions are related to states 1, 2, 3, and 4, respectively, with probability values of 0.2, 0.2, 0.3, and 0.3, respectively. In this study, for parameters with cognitive uncertainty, PFNs are used, which have a uniform distribution, in such a way that for each parameter, 5 numbers are randomly selected in the range of uniform distribution, and the PFN is identified for each parameter. The range of the uniform distribution of uncertain parameters is shown in Table 2.

In the Appendix, we describe the IFPS approach method for solving the bi-objective model. The suggested problem is solved by using GAMS 24.8 software (*https://www.gams.com/products/gams/gams-language*) and the CPLEX solver (*https://www.ibm.com/products/ilog-cplex-optimization-studio/cplex-optimizer*), and the performance of the model is examined in terms of its robustness and sensitivity. In the following, the model is assessed using simulation and the production of nominal data through the realization model.

6.3. Robustness analysis

This part performs the robustness analysis of the proposed RSP programming model. Analysis of robustness coefficients, including possibilistic deviation, scenario deviation, and un-fulfilment of demand and capacity, are investigated and analysed by changing coefficients ¥, Φ, θ, and π.

6.3.1. Possibilistic deviation

The RSP programming with a focus on credibility measures is examined in this analysis. Figure 5 indicates the impact of the coefficient ¥ on the possibilistic deviation. In the case where ¥ = 0, the risk is at the most heightened value, the possibilistic deviation value is at the maximum, and the mean cost is at the lowest value. By increasing the ¥ coefficient, the decision risk and possibilistic deviation are reduced, and the mean cost will increase due to the robustness of the optimality.

The optimality robustness can be managed by changing the importance of the possibilistic deviation. Due to DMs' preferences and level of risk-taking, the coefficient ¥ can be changed to provide different answers according to different conditions and acceptable risk levels of DMs. Therefore, the proposed approach has the appropriate flexibility to determine solutions by changing the possibilistic deviation coefficient.

6.3.2. Scenario deviation

The findings of changing the coefficient Φ are illustrated in Figure 6. When Φ = 0, the mean of the objective functions is at its lowest point, and the risk and scenario deviations are at their maximum points. The mean of the objective functions rises when the coefficient Φ is increased during the decision risk and scenario deviation decrease.

As a result, altering the scenario deviation coefficient can be used to control the optimality robustness. In the proposed approach, by changing the Φ, it is possible to achieve flexibility solutions in different conditions and based on the level of risk of DMs. Based on this, it is possible to create a trade-off by changing the coefficient Φ, between the mean of the objective functions and the risk of DMs.

Table 2. Range of uniform distribution for uncertain parameters

Parameter (unit)	Scenarios				Unit
	State 1	State 2	State 3	State 4	
	0.2	0.2	0.3	0.3	
$\tilde{d}_{qcts}\,(q=1)$	$U(155-275)$	$U(165-305)$	$U(175-335)$	$U(185-365)$	ton
$\tilde{d}_{qcts}\,(q=2)$	$U(185-355)$	$U(205-395)$	$U(225-435)$	$U(245-475)$	ton
$\tilde{d}_{qcts}\,(q=3)$	$U(215-405)$	$U(255-445)$	$U(285-485)$	$U(315-525)$	ton
$\tilde{d}_{uvt}$	$U(85-215)$				ton
$\widetilde{pp}_m$	$U(17000-28000)$				ton
$\widetilde{pp}_d$	$U(11000-20000)$				ton
$\widetilde{pp}_l$	$U(10000-19000)$				ton
$\widetilde{pp}_g$	$U(11000-20000)$				ton
$\widetilde{pp}_n$	$U(9000-20000)$				ton
$\widetilde{co}_{qmdts}$	$U(105-555)$	$U(115-615)$	$U(125-675)$	$U(135-735)$	$\cdot 10^3$ toman
$\widetilde{cu}_{qdcts}$	$U(125-725)$	$U(135-795)$	$U(145-865)$	$U(155-935)$	$\cdot 10^3$ toman
$\widetilde{cq}_{qclts}$	$U(125-725)$	$U(135-795)$	$U(145-865)$	$U(155-935)$	$\cdot 10^3$ toman
$\widetilde{cp}_{qlgts}$	$U(105-555)$	$U(115-615)$	$U(125-675)$	$U(135-735)$	$\cdot 10^3$ toman
$\widetilde{cs}_{qlnts}$	$U(105-555)$	$U(115-615)$	$U(125-675)$	$U(135-735)$	$\cdot 10^3$ toman
$\widetilde{ch}_{qgdts}$	$U(105-555)$	$U(115-615)$	$U(125-675)$	$U(135-735)$	$\cdot 10^3$ toman
$\widetilde{cv}_{unvts}$	$U(105-605)$	$U(115-665)$	$U(125-725)$	$U(135-785)$	$\cdot 10^3$ toman
$\tilde{\rho}_{qmts}$	$U(205-605)$	$U(225-665)$	$U(245-725)$	$U(265-785)$	$\cdot 10^3$ toman
$\tilde{\pi}_{qdts}$	$U(155-255)$	$U(175-275)$	$U(195-295)$	$U(215-315)$	$\cdot 10^3$ toman
$\tilde{\partial}_{qlts}$	$U(155-255)$	$U(175-275)$	$U(195-295)$	$U(215-315)$	$\cdot 10^3$ toman
$\tilde{\tau}_{qgts}$	$U(205-405)$	$U(245-445)$	$U(285-485)$	$U(325-525)$	$\cdot 10^3$ toman
$\tilde{\gamma}_{unts}$	$U(205-405)$	$U(245-445)$	$U(285-485)$	$U(325-525)$	$\cdot 10^3$ toman
$\widetilde{fc}_m$	$U(1800000-24000000)$				$\cdot 10^3$ toman
$\widetilde{fc}_d$	$U(9000000-12000000)$				$\cdot 10^3$ toman
$\widetilde{fc}_l$	$U(8000000-11000000)$				$\cdot 10^3$ toman
$\widetilde{fc}_g$	$U(10000000-14000000)$				$\cdot 10^3$ toman
$\widetilde{fc}_n$	$U(12000000-18000000)$				$\cdot 10^3$ toman
$\tilde{e}_{qmts}$	$U(14-26)$	$U(20-37)$	$U(26-38)$	$U(32-49)$	g
$\tilde{e}_{qdts}$	$U(6.5-16.5)$	$U(7.5-17.5)$	$U(8.5-18.5)$	$U(9.5-19.5)$	g
$\tilde{e}_{qlts}$	$U(6.5-16.5)$	$U(7.5-17.5)$	$U(8.5-18.5)$	$U(9.5-19.5)$	g
$\tilde{e}_{qgts}$	$U(6-19)$	$U(7-20)$	$U(8-21)$	$U(9-22)$	g
$\tilde{e}_{unts}$	$U(7-20)$	$U(8-21)$	$U(9-22)$	$U(10-23)$	g
$\tilde{e}_{qmdts}$	$U(45-175)$	$U(65-195)$	$U(85-215)$	$U(105-235)$	g
$\tilde{e}_{qdcts}$	$U(60-210)$	$U(80-230)$	$U(100-250)$	$U(125-270)$	g
$\tilde{e}_{qclts}$	$U(60-210)$	$U(80-230)$	$U(100-250)$	$U(125-270)$	g
$\tilde{e}_{qlgts}$	$U(45-175)$	$U(65-195)$	$U(85-215)$	$U(105-235)$	g
$\tilde{e}_{qlnts}$	$U(45-175)$	$U(65-195)$	$U(85-215)$	$U(105-235)$	g
$\tilde{e}_{qgdts}$	$U(45-175)$	$U(65-195)$	$U(85-215)$	$U(105-235)$	g
$\tilde{e}_{unvts}$	$U(60-210)$	$U(80-230)$	$U(100-250)$	$U(125-270)$	g
$\tilde{w}_{qc}$	$U(15-45)$				%
$\tilde{\omega}_{qt}$	$U(20-35)$				%

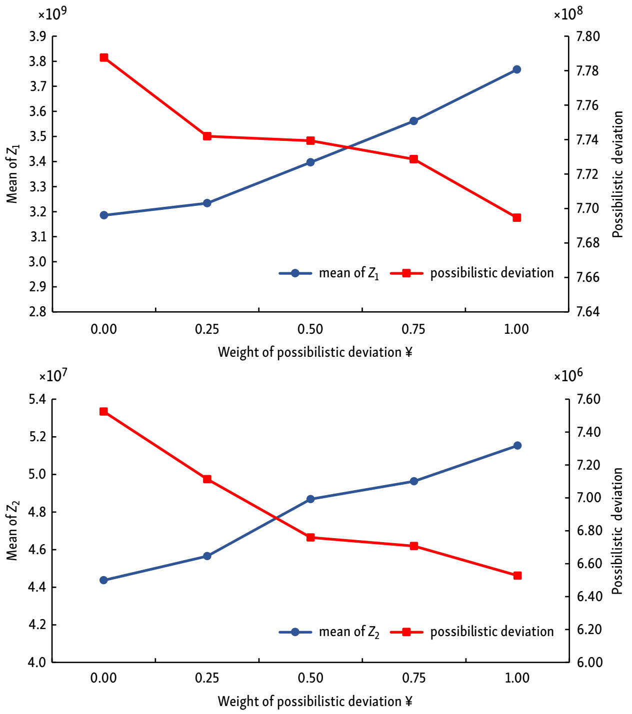

Figure 5. Robustness analysis of possibilistic deviation

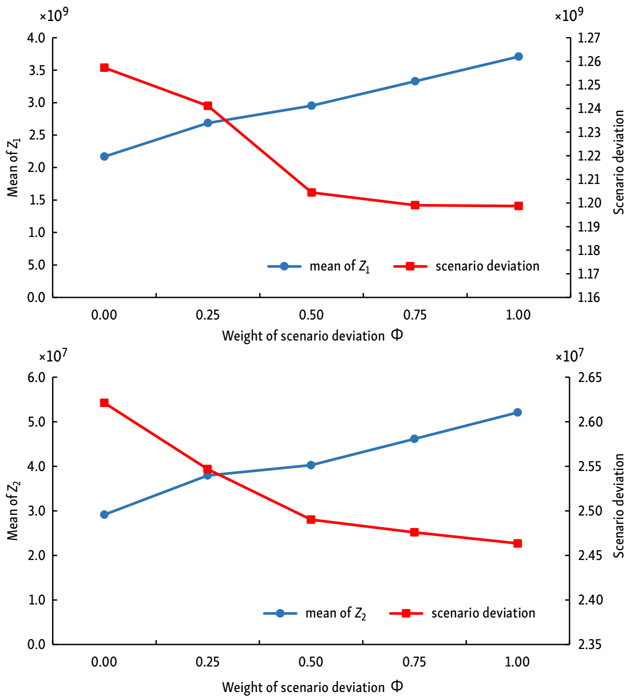

Figure 6. Robustness analysis of scenario deviation

6.3.3. Un-fulfilment of constraints

This section examines the impact of changing coefficients θ and π on the mean cost in the RSP programming model based on credibility measures. In Figures 7 and 8, the robustness analysis of changing coefficients θ and π are presented.

When the coefficients θ and π are 0, the mean cost is at the lowest level. The feasibility robustness increases with the penalty coefficient θ and π, and the risk decreases. Therefore, by changing the coefficients θ and π, a trade-off is made according to the risk of DMs and the mean cost. The robustness of the feasibility of the solutions is handled by determining the coefficients regarding the priorities of DMs, and the presented method is suitable for selecting a set of solutions by varying the coefficients θ and π in the various conditions.

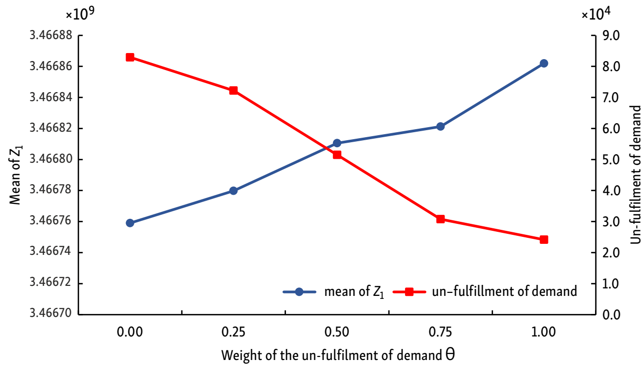

Figure 7. Robustness analysis of un-fulfilment demand

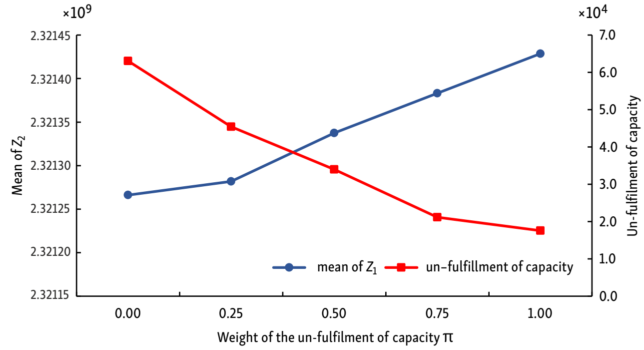

Figure 8. Robustness analysis of un-fulfilment capacity

6.4. Sensitivity analysis

The sensitivity analysis of the facility opening cost, facility capacity, demand, and product failure rate is analysed, and the effects are analysed on the average of objective functions.

6.4.1. Facility opening costs

Facility opening costs impact the number of active facilities. This study examines the influence of facility opening costs on average costs and carbon emission levels from a 20% drop to a 20% rise. Figure 9 shows the sensitivity analysis of the facility opening cost on the average objective functions. The average cost increases with the increase in the cost of opening the places, and the average cost decreases with the decrease in the opening cost. The reason for this is the direct relationship between the mean of costs and the cost of opening the facility. Also, with the increase in the cost of opening the places, fewer places are utilized, and as a result, the distance among the places increases, and the amount of carbon emission increases due to transportation. By reducing the cost of opening, more places are utilized, and as a result, the distance among locations is decreased, and carbon pollution is reduced due to transport. Therefore, at different levels of the costs of opening, it is possible to control and manage the total costs and the carbon emissions at an appropriate level.

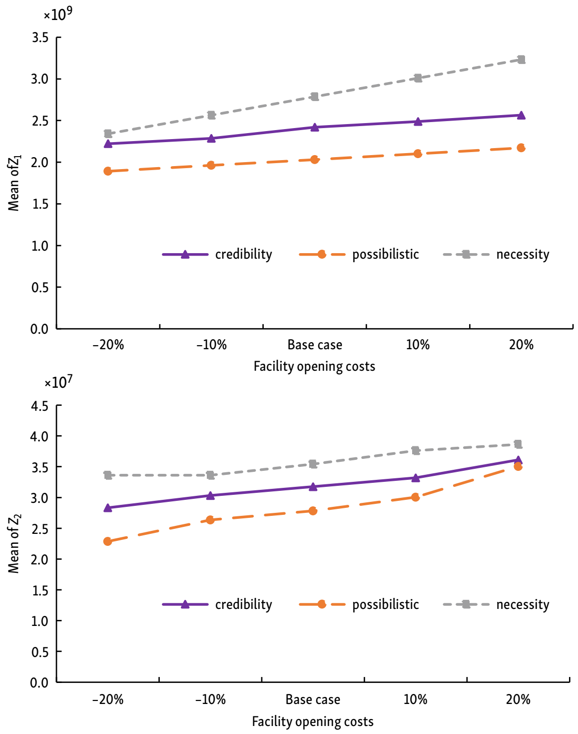

Figure 9. Sensitivity analysis of facility opening cost

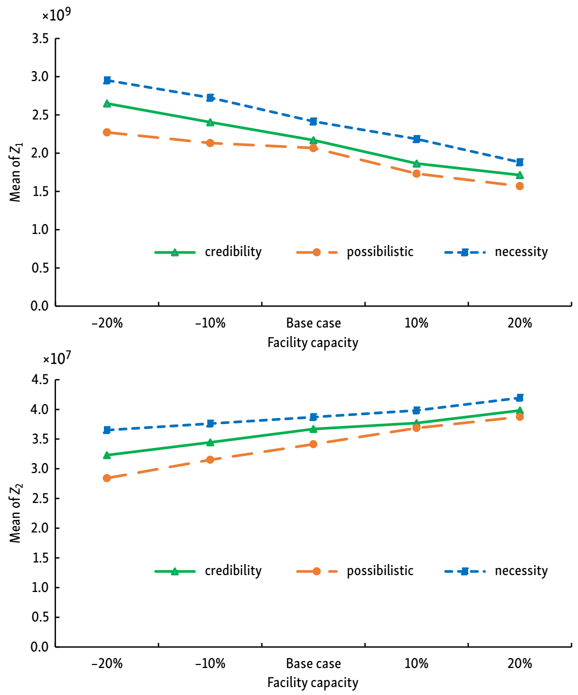

Figure 10. Sensitivity analysis of facility capacity

6.4.2. Facility capacity

The influence of facility capacity on average cost and carbon emissions is examined in this section. In this investigation, the influence of facility capacity on average objective functions is investigated in the range of a 20% drop to a 20% rise, which the outcomes are shown in Figure 10. Increasing the facility capacity means fewer sites are activated, and the total costs decrease. By reducing the capacity to meet the demand, activating more facilities in the model is necessary, increasing the average costs. On the other hand, with the reduction of facility capacity, more sites are required to handle the demand, and as a result, the distance between different locations is reduced, which decreases the carbon emissions by transportation. Therefore, appropriately controlling and managing the total costs and carbon emissions is possible.

6.4.3. Demand

In the design of SC, demand is crucial. In the model, fewer facilities are activated when demand is low, and more places are active when demand is high. Therefore, an increase in demand leads to a growth in the number of active facilities and, as a result, an increase in fixed costs and processing costs, which shows a growth in the mean cost, and with a decrease in demand, fewer facilities are needed, which leads to a decrease in the average cost. Carbon emissions also rise when demand rises due to increasing processes at various sites and transit between sites. Therefore, total costs and carbon emissions can be managed appropriately at different demand levels. Figure 11 indicates the findings of the sensitivity analysis of demand.

6.4.4. Product return rate

The sensitivity analysis of the product return rate is discussed in this part because it impacts processing and transportation in the facilities and is a critical parameter in the SC design. The product return rate's sensitivity analysis is shown in Figure 12. According to the presented results, with the increase in the product return rate, more facilities are required, and the transportation and processing of goods expands. As a result, an increase in the return rate should increase overall expenses and carbon emissions from processing and transportation, and vice versa. Therefore, according to the presented results, costs and emissions can be handled at different levels of product return rates, and production quality can be controlled appropriately. The appropriate cost and carbon emissions level are achieved based on DMs' opinions.

6.5. Performance assessment of the proposed approaches

The results are simulated through nominal data to assess the RSP programming procedure based on credibility measures. For per ambiguous parameter, random data is produced with a uniform distribution, and the realization model is then applied to validate the proposed models; by evenly producing ten random realizations, the proposed models are assessed.

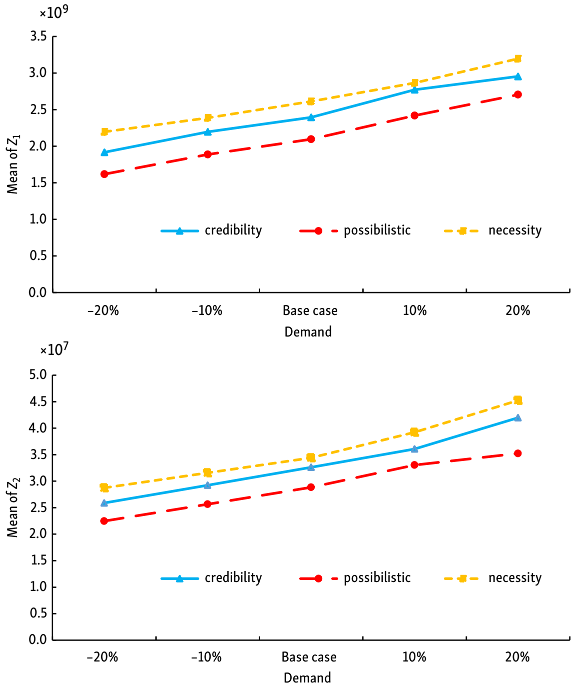

Figure 11. Sensitivity analysis of demand

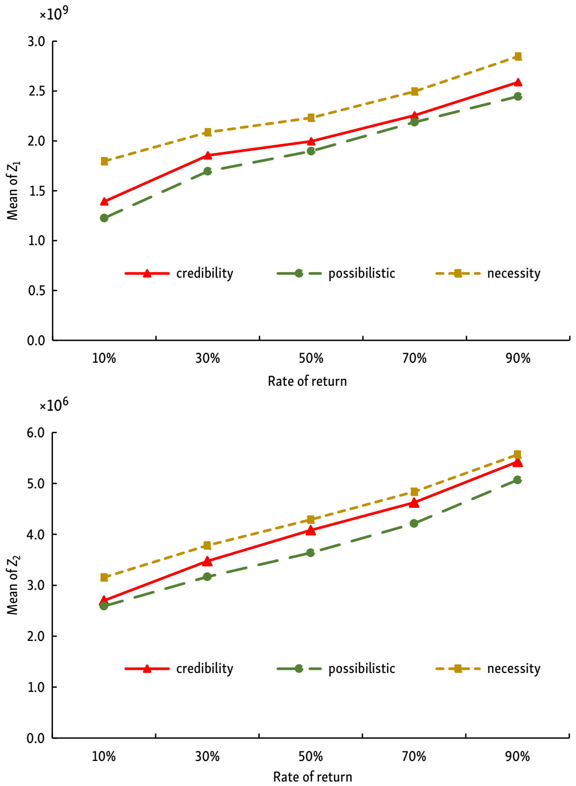

Figure 12. Sensitivity analysis of product return rate

Assume the parameter $\tilde{£} = (£_1, £_2, £_3, £_4, £_5)$ is considered a PFN. A number is generated randomly per run in the range $(£_1, £_5)$. In the following, the obtained answers are examined under nominal data. At this stage, the obtained answers from the presented models under the data (x^*, y^*) are replaced in the model. The model can be expressed in the following compact form:

$$\max Z^{real} = €_1 \cdot \frac{z_1^{NIS^*} - \left(f^{real} \cdot y^* + c^{real} \cdot x^*\right)}{z_1^{NIS^*} - z_1^{PIS^*}} + €_2 \cdot \frac{z_2^{NIS^*} - \left(em^{real} \cdot x^*\right)}{z_2^{NIS^*} - z_2^{PIS^*}} - \left(\zeta_1 \cdot S_d^r + \zeta_2 \cdot S_c^r\right)$$

subject to:

$$A \cdot x^* + S_d^r \geq d^{real};$$
$$B \cdot x^* = 0;$$
$$S \cdot x^* \leq N^{real} \cdot y^* + S_c^r;$$
$$T \cdot y^* \leq 1;$$
$$S_d^r,\ S_c^r \geq 0. \quad (69)$$

S_d^r and S_c^r indicate constraint violation degree, while ζ_1 and ζ_2 indicate the penalty for violations. The realization model calculates and compares the objective's average and SD for the proposed models. Normalizing the penalty costs for chance constraints has resulted in a significantly lower average. So, the mean and SD of the chance constraints to compare and assess the presented model are used. The SPP and RSP approaches are compared based on credibility measures at different confidence levels. The numerical findings and evaluation of the suggested SPP and RSP approaches are presented in Table 3.

The performance evaluation results of the approaches showed that the values lowest of average and SD related to the RSP approach at the confidence level of 0 ≤ α, β ≤ 0.25, and the values highest of average and SD related to the SPP approach at the confidence level Confidence is α, β = 0.8. RSP approaches have lower values of average and SD due to consideration of robustness and control of possibilistic and scenario deviations and un-fulfilment of demand and capacity compared to SPP approaches. Therefore, considering robustness increases the accuracy and improves the solutions in PFNs. Also, with the increase in confidence levels, the solutions get worse due to the shrinking of the justified space, and as a result, the average and SD increase with the increase of the confidence level. Therefore, for the confidence level of 0 ≤ α, β ≤ 0.25, the RSP approach has the best performance, and the solutions of the RSP approaches become worse as the confidence level values increase. In SPP approaches, the best answer was related to the confidence level of α, β = 0.2, which worsened the average and SD values with the increase of the confidence level. The outcomes of the numerical simulation demonstrated that the RSP model performs better than the SPP model based on PFNs.

6.6. Discussion and managerial insights

This study provides a framework for managers to consider hybrid uncertainty in sustainable CLSCND, which can consider both fuzzy and robust aspects. The proposed approach considers the maximum subjectivity of DMs compared to triangular and trapezoidal fuzzy numbers. Using PFNs, SC managers can consider more uncertainty of non-deterministic parameters, and DMs lose less information by using the proposed approach. In addition, decision-

Table 3. The numerical findings of the SPP and RSP approaches

No	SPP approach				RSP approach			
	α, β = 0.2	α, β = 0.4	α, β = 0.6	α, β = 0.8	0 ≤ α, β ≤ 0.25	0.25 ≤ α, β ≤ 0.5	0.5 ≤ α, β ≤ 0.75	0.75 ≤ α, β ≤ 1
1	5807	6778	7411	9316	4710	4696	4972	5107
2	5125	6894	8309	9071	4720	4699	4922	5296
3	5479	6392	8005	10816	4728	4842	4892	5118
4	5796	7392	8492	10022	4729	4842	4901	5115
5	4900	7199	10258	11522	4775	4916	4811	4941
6	5351	7533	8583	11378	4496	4854	5047	5641
7	5798	7467	9376	10534	4781	4892	5183	5030
8	5033	7600	9670	10190	4774	4880	5118	5469
9	5301	7934	9964	12846	4671	5049	5254	5107
10	5988	7668	8679	10503	4696	4857	5190	5246
Average	5458	7286	8875	10620	4708	4853	5029	5207
SD	357	446	864	1051	78	97	143	201

making is provided more accurately. Therefore, uncertainty is significantly considered by using PFNs in modelling, and the presented results for managers are accurate.

In addition, this study presents a novel RSP programming based on PFNs to solve the SCLSC problems under hybrid uncertainty and improve the degree of ambiguity and lack of information for SC managers. The proposed robust approach presents different ranges of consideration of DMs' preferences and managers' risk tolerance based on PFNs. In this study, the possibility measure was used to consider the risk-taking state, and the necessity criterion was used to consider the risk-aversion state in the preferences of DMs. Credibility measure was also used to eliminate the weaknesses of optimistic and pessimistic spectrums and to present the intermediate state of managers' risk-taking. The model uses the credibility measure to dynamically apply the risk-taking viewpoints of DMs. This benefit enables managers to make decisions in SCLSC modelling that are more flexible, and on the other hand, because of the model's robustness, the minimum level of confidence is optimally determined through the model's solution, and the drawback of SPP was corrected that necessitated frequent checks by DMs. In addition, the proposed RSP programming model achieves a desirable satisfaction degree, and manager judgments are more exact than earlier procedures using a range of pentagonal numbers for uncertain parameters.

A study was conducted to decrease costs and carbon pollution in the stone paper SC. The promotion and feasibility of stone paper product development led to creating a perspective for managers and policymakers to produce these clean products due to diverse applications, no need for water, and no environmental pollution. The analysis's findings demonstrated that SC managers could use the suggested method to alter the robustness coefficient to make trade-offs among the degree of risk and mean of objective functions.

The proposed approach provides realistic and flexible solutions for managers under different conditions of uncertainty based on pessimistic-optimistic preferences and at different risk tolerance levels for managers in the stone paper area. In addition, the outcomes of the sensitivity analysis of the parameters revealed that at different levels of facility opening cost, facility capacity, demand, and product return rate in the studied SC, costs and carbon emissions can be controlled at an appropriate level based on the opinions of DMs. The simulation results showed that considering optimality and feasibility, robustness reduced risk and presented more reliable and accurate results than SPP models for sustainable CLSCND. Using robust approaches led to minimizing the sensitivity of changing parameters in conditions of uncertainty and risk and obtaining more realistic and accurate solutions.

7. Conclusions

This study used a novel RSP programming approach to examine the SCLSC problem under hybrid uncertainty. The studies used triangular and trapezoidal fuzzy numbers to model SCLSC. PFNs had higher accuracy due to considering the maximum subjectivity of experts, providing greater freedom of action for DMs, considering higher uncertainty, and lacking less information. PFNs were more comprehensive and accurate and could be converted to triangular and trapezoidal sets based on the values of the membership degree coefficient. As a result, to solve the existing deficiencies, a novel stochastic possibilistic approach was developed based on PFNs. According to the weaknesses of SPP, a novel RSP programming approach was developed for PFNs based on the necessity, possibility, and credibility measures. In the proposed robust approach, solving the model determined the minimum confidence level, and repeated mental checks were eliminated to find the appropriate confidence level. A study was conducted to develop a stone paper SCLSC in Iran. The implementation results demonstrated that the proposed approach trades-offs between the mean of objectives and risk by modifying the robustness coefficients. Also, in the proposed approach, by considering the uncertainty through PFNs, realistic and flexible approaches can be provided in different conditions of uncertainty based on different levels of risk of DMs through necessity, possibility, and credibility measures. Considering robustness led to the achievement of solutions with low sensitivity to changing parameters, which reduced the risk of long-term decisions in conditions of hybrid uncertainty. The effectiveness of the suggested RSP approach was also confirmed through numerical simulation.

One of the limitations of the present study was considering only the cognitive and random uncertainties, while in some SC network configuration problems, some constraints are flexible. Flexible programming can be considered in the modelling based on PFNs as a future research field. In this research, robust linear programming was expanded for the SC design problem, but for problems with large dimensions and non-linear problems, researchers can pay attention to the development of heuristic and meta-heuristic approaches as a new research field.

Disclosure statement

Authors do not have any competing financial, professional, or personal interests from other parties.

Appendix. IFPS

The IFPS approach is considered because of its high flexibility and ability to determine how well each objective functions. Based on the importance of each objective, this method enables DMs to choose the ideal option (Günay *et al.* 2021). The TH method is one of the methods of the IFPS approach; the phases of the approach are as follows (Torabi, Hassini 2008):

- ***Phase 1***: a NIS and a PIS were specified;
- ***Phase 2***: then the membership function is computed:

$$Z_1^{PIS} = \min Z_1;\quad Z_1^{NIS} = \max Z_1;\quad Z_2^{PIS} = \min Z_2;\quad Z_2^{NIS} = \max Z_2; \tag{70}$$

$$\mu_{Z_1} = \begin{cases} 1, & Z_1 < Z_1^{PIS}; \\ \dfrac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}}, & Z_1^{PIS} \le Z_1 \le Z_1^{NIS}; \\ 0, & Z_1 > Z_1^{NIS}; \end{cases} \tag{71}$$

$$\mu_{Z_2} = \begin{cases} 1, & Z_2 < Z_2^{PIS}; \\ \dfrac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}, & Z_2^{PIS} \le Z_2 \le Z_2^{NIS}; \\ 0, & Z_2 > Z_2^{NIS}; \end{cases} \tag{72}$$

- ***Phase 3***: in this phase, the aggregation function is determined:

$$\max \S(x) = \sigma \cdot \lambda_0 + (1-\sigma) \cdot \sum_{h=1}^{2} \varphi_{Z_h} \cdot \mu_{Z_h}(x)$$

subject to:

$$\lambda_0 \le \mu_{Z_h}(x),\ h = 1, 2;\quad x \in F(x);\quad \lambda_0, \sigma \in [0, 1], \tag{73}$$

where: $F(x)$ is the feasibility range of the deterministic multi-objective model; λ_0, $\mu_{Z_h}(x)$ present the minimum and degree of satisfaction with the hth objective, respectively; φ_{Z_h} is the weight of the hth objective according to the conditions $\sum_h \varphi_{Z_h} = 1$ and $\varphi_{Z_h} > 0$. The compromise coefficient is determined through the parameter σ, which regulates the degree of compensation between goals and the minor satisfaction grade;
- ***Phase 4***: the value of the parameter σ and φ_{Z_h} is determined and replaced in the Model (73), and then the model is solved;
- ***Phase 5***: if the DMs are comfortable, the process is prevented, and otherwise, the DMs modify the σ and φ_{Z_h} and then the process is repeated from Phase 3.

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