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A MULTICRITERIAL MATHEMATICAL MODEL FOR CREATING VEHICLE ROUTES TO TRANSPORT GOODS

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Abstract. The article describes modelling a vehicle route to transport goods and discusses models for determining vehicle routes for a single depot and more vehicles. Above all, the paper defines a mathematical model for determining a vehicle route for more depots, more vehicles and more types of transport requirements.

Keywords: goods transport, more depots, more vehicles, operations research, route.

1. Introduction

Complete logistics involves movements of goods from the transportation of raw materials, through various types of storage, distribution of products to the consumer, to the final disposal of goods at the end of their useful life. The routes of goods transport with maximum efficiency (minimizing travel distance, minimum costs of transport, the minimum duration of transport, the minimum number of vehicles, maximum vehicle utilization) should be proposed at all stages of this process.

When designing freight transport routes, a traffic controller has to minimize vehicle running costs. One of the possible reductions is a suitable organization of vehicle assignments on various transportable requirements. In such a case, various possibilities can be used – one depot (more depots), one vehicle (more vehicles) and one kind of transported goods – requirement (more kinds of transported requirements).

This article introduces a mathematical model for more depots, more vehicles and more types of transport requirements.

2. The Problem of Determining a Vehicle Route

The tasks that attempt to determine an optimal route for each vehicle in a way that the specified conditions are observed (return to the starting point also belongs to these conditions) from the given parameters (location of each object to be served, distance, cost of moving between locations, the number and location of depots, the number of vehicles available, the capacity of individual vehicles, the times when each site must be served, etc.) are called gyratory traffic tasks.

Gyratory traffic tasks belong to the category of optimization tasks on graphs dealing with a mathematical discipline called *graph theory*, described in Diestel (2006) and Zykov (Зыков 2004). The optimization problem of gyratory traffic tasks in researching operations is identified as VRP (Vehicle Routing Problem) and is listed, for example, in Morse and Kimball (2003); Dudorkin (2002). VRP can be sorted according to the number and requirements for the customers of the following categories:

- static – the number of customers and their demands (requirements) are known in advance;
- dynamic – after the departure of a vehicle, controllers receive additional requirements.

To simplify the VRP solution, a task in a travelling salesman (Travelling Salesman Problem – TSP) can be used. The aim is to leave the depot, visit all customers exactly once and return to the starting depot, so that the travelled distance is minimal. The presented method is used for one depot and described in Tuzar and Pastor (2007). The Travelling Salesman Problem can be solved as:

- symmetrical;
- metrical;
- Euclidean;
- open (the vehicle is not returned to the starting depot);
- with time intervals.

By modifying VRP, you attain a capacitive limited task on vehicle routing (Capacitated Vehicle Routing Problem – CVRP) which is a combination of the travelling salesman problem (TSP) and loading problem (Bin Packing Problem – BPP). The algorithm of the method is listed in Ralphs *et al.* (2003).

The problems determining optimal routes have three basic levels:

- strategic level deal with the number, type and distribution of the attended places. An example of a solution to warehouses with linkage to their operation is listed in Baublys (2008);
- tactical level determines the number of necessary operational elements and is theoretically shown in Jablonský (2001). In case of freight transport, the required number of vehicles, their types and capacity as well as the number of drivers are listed in Kleprlík *et al.* (1999);
- operational level seeks specific route scheduling options, sets schedules and duty rosters for each driver. Creating duty rosters is described in Kleprlík (2007). The aim of this article is determining vehicle routes for more depots, more vehicles and more types of transport requirements.

The determination of optimal routes is affected, in addition to the above mentioned three levels, by the cost of each type of transport or multimodal transport. An example of a mathematical model is presented in Lingaitienė (2008).

An assignment on a vehicle route represents the so-called 'multiple problem of a commercial traveller'. When solving this case, the use of more vehicles is assumed ('commercial travellers'). Commercial travellers are supposed to visit all attended places only once. We further assume that each of these places has a specific transport requirement (how many goods the place needs) – a rolling stock of vehicles going out and coming back to the same depot is available; a number of depots is higher than one; each of the vehicles has to visit at least one pinnacle in a route; task limitation results from the limited weight capacity of vehicles. Moreover, conveying time regarding a linkage to the transit period may also be a time limiting factor.

3. Determination of a Vehicle Route – One Depot, More Vehicles

In this case, we minimize total running performance (total distance covered), incurred costs or transit period. Deterministic demands for each pinnacle and capacity weight of each vehicle are also defined. The aim is to determinate a transport route for each vehicle so that to minimize a criterion. Then, transport demand for each pinnacle would be satisfied and vehicle capacity would not be exceeded (for example, see Fig. 1).

4. Determination of a Vehicle Route – More Depots, More Vehicles

It is almost the same case as the previous one discussed in Chapter 2 with the only difference in the number of depots. More depots with various numbers of vehicles are available. Each vehicle has to return to the same depot from which it was dispatched (see Fig. 2).

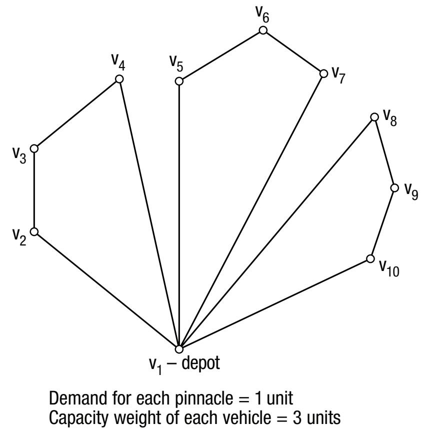

Fig. 1. One depot, more vehicles

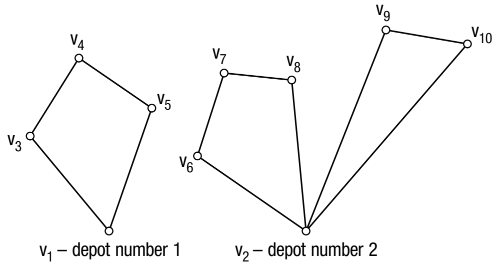

Fig. 2. More depots, more vehicles

5. Determination of a Vehicle Route – One Depot, More Vehicles, Stochastic Demand

The case is similar to the one discussed in Chapter 2 with the exception that demand in each pinnacle is unknown; however, it stems from a stochastic (probability) division.

6. Mathematical Model for Determining a Vehicle Route – More Depots, More Vehicles, More Types of Transport Requirements

A defined number of vehicles is supposed to visit n pinnacles of the transport network in a way that the distance covered by all vehicles (costs, total convey time) is minimal. Vehicles can be garaged in different depots; each vehicle has to return to the same depot from which it was dispatched. A vehicle of type k is necessary to be used for transport requirement of type k. The capacity of vehicles for transporting the same type of requirements is the same. Demands for a single pinnacle are accommodated only by one vehicle and each pinnacle with requirement k is only once attended by a vehicle of type k.

Used marks:

n – the number of operated pinnacles (tops);
p – the number of different types of requirements;
M – the number of different types of depots;
M_k – the number of depots of type k;
A – the total number of vehicles;
a^k – the number of vehicles of type k;

K_v^k – the capacity of vehicle v for transport requirement of type k;
T_v^k – maximum allowed time for a route of vehicle v of type k;
d_i^k – the size of type k requirement in pinnacle i;
t_i^{vk} – time needed for vehicle v of type k to load and unload in pinnacle i;
t_{ij}^{vk} – driving time of vehicle v of type k from pinnacle i to pinnacle j ($t_{ii}^{vk} = \infty$);
c_{ij} – travelling expenses from pinnacle i to pinnacle j.

We put $x_{ij}^{vk} = 1$, if vehicle v operates requirement k along the route from pinnacle i to pinnacle j; otherwise $x_{ij}^{vk} = 0$.

We specify depots as pinnacles with index numbers 1, ..., M. The operated pinnacles will therefore have index numbers M + 1, ..., n + M. We minimize functions 1 in compliance with restrictive conditions 2÷9:

$$\sum_{i=1}^{n+M}\sum_{j=1}^{n+M}\sum_{v=1}^{a^k} c_{ij} x_{ij}^{vk} \to \min \quad \text{for } k = 1, ..., p;$$
$$a^k \to \min \quad \text{for } k = 1, ..., p. \qquad (1)$$

Restricting conditions:

$$\sum_{i=M+1}^{n+M}\sum_{v=1}^{a^k} x_{ij}^{vk} = 1 \quad \text{for } j = M+1, ..., n+M, \quad \text{for } k = 1, ..., p; \qquad (2)$$

one vehicle only drives into each operated pinnacle:

$$\sum_{j=M+1}^{n+M}\sum_{v=1}^{a^k} x_{ij}^{vk} = 1 \quad \text{for } i = M+1, ..., n+M, \quad \text{for } k = 1, ..., p; \qquad (3)$$

one vehicle only leaves each operated pinnacle:

$$\sum_{i=1+M}^{n+M} x_{il}^{vk} - \sum_{j=1+M}^{n+M} x_{lj}^{vk} = 0 \quad \text{for } v = 1, ..., a^k, \quad \text{for } l = 1+M, ..., n+M, \quad \text{for } k = 1, ..., p; \qquad (4)$$

a vehicle has to leave the pinnacle to which it drove in:

$$\sum_{i=1+M}^{n+M} d_i^k \left(\sum_{j=1+M}^{n+M} x_{ij}^{vk} \right) \le K_v^k \quad \text{for } v = 1, ..., a^k, \quad \text{for } k = 1, ..., p; \qquad (5)$$

conditions for capacity weight of vehicles:

$$\sum_{i=1}^{n+M} t_i^{vk} \sum_{j=1}^{n+M} x_{ij}^{vk} + \sum_{i=1}^{n+M}\sum_{j=1}^{n+M} t_{ij}^{vk} x_{ij}^{vk} \le T_v^k \quad \text{for } v = 1, ..., a^k, \quad \text{for } k = 1, ..., p; \qquad (6)$$

conditions for the total time:

$$\sum_{i=1}^{M}\sum_{j=M+1}^{n+M} x_{ij}^{vk} \le 1 \quad \text{for } v = 1, ..., a^k, \quad \text{for } k = 1, ..., p; \qquad (7)$$

vehicle v of type k drives from depot i to depot j once at a maximum:

$$\sum_{i=1}^{M}\sum_{j=M+1}^{n+M} x_{ji}^{vk} \le 1 \quad \text{for } v = 1, ..., a^k, \quad \text{for } k = 1, ..., p; \qquad (8)$$

vehicle v of type k drives from pinnacle j to depot i once at a maximum:

$$\sum_{i \in Q}\sum_{j \notin Q} x_{ij}^{vk} \ge 1 \quad \text{for } v = 1, ..., a^k, \quad \text{for } k = 1, ..., p. \qquad (9)$$

Q is an arbitrary subset of graph pinnacles that includes all depots. A route has to exist from set Q to set $\overline{Q}$.

Criteria limiting the final form of transportation routes are shown in Table.

Table. Criteria limiting the final form of transportation routes

Character of demand	deterministic; stochastic
Location of demand	only in a traffic junction (in terminals, by sender)
Operation in junction	only loading; only unloading; loading and unloading
Kind of a network	directed; undirected; mixed
Size of rolling stock	one vehicle; more vehicles
Structure of rolling stock	homogenous (only one type of vehicle); heterogeneous (several types of vehicles)
Capacity of vehicles	for all vehicles equal; for different types of different vehicles; odd
Parking of vehicles	only one depot; some depots; parking in the place of a driver's residence
Costs	fixed costs; variable costs
Objective function	maximisation of benefit for sender; minimisation number of vehicles; minimisation of vehicle running costs; minimisation of the total distance covered

7. Conclusions

Transporters' effort is to minimize vehicle running costs (operating costs). One of possible ways is a suitable organization of vehicle assignments to transportable requirements and working out routes. The mathematical model introduced in the article focuses on the case when different kinds of requirements are brought together to depots (or distributed from depots). This model could be applied to solve a practical problem of distributing goods from manufacturers to warehouses or from warehouses to shops. The described model can be also used in other section of logistic chains where we hand out (bring together) various types of goods from one or several different depots.

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